

③ PT. $(w^R)^R = w \quad \forall w \in \Sigma^*$

(Induction)

$$(uw)^R = u^R w^R$$

(n-1) length: $u = wa$

$$(uwa)^R = (wa)^R u^R$$

$$= aw^R u^R$$

\therefore true $\forall w \in \Sigma^*$

④ $L = \{ab, aa, baa\}$

which of the following strings are in L^*, L^+ ?

abaabaabaa L^+, L^*

aaaabaaaa L^+, L^*

baaaaaabaaaaab

baaaaaabaa L^+, L^*

⑤ let $\Sigma = \{a, b\}$ Use set notation to describe \bar{L} .

$L = \{aa, bb\}$

$\bar{L} = U - \{aa, bb\}$

$L = \{\lambda, a, b, ab, ba\} \cup \{w : |w| > 2, w \in \Sigma^*\}$

⑥ let L be any language on a non-empty alphabet. show that L, \bar{L} cannot be both finite.

Case (i) L is finite

\rightarrow we know U is infinite

$\Rightarrow \bar{L} = U - L$

= infinite lang - finite lang

= infinite lang.

Case (ii) L is infinite

U is infinite

$\bar{L} = U - L$

= finite

From above, in any case, both cannot be finite

⑦ Are there any languages for which $\overline{L^*} = (\overline{L})^*$?

$$\lambda \in L^*$$

$$\Rightarrow \lambda \notin \overline{L^*}$$

but $(\overline{L})^*$ contains λ

\therefore No language satisfies $\overline{L^*} = (\overline{L})^*$

⑧ Pt. $(L_1 L_2)^R = L_2^R L_1^R \quad \forall L_1, L_2$

$$\text{let } u \in L_1, v \in L_2$$

$$L_1 L_2 \Rightarrow uv$$

$$(L_1 L_2)^R \Rightarrow (uv)^R$$

$$= v^R u^R = L_2^R L_1^R \quad \forall u, v.$$

$$\therefore (L_1 L_2)^R = L_2^R L_1^R \quad \forall L_1, L_2$$

⑨ Show that $(L^*)^* = L^* \quad \forall L$.

$$\text{let } \Sigma = \{a, b\}$$

$$L^* \Rightarrow \Sigma^* \Rightarrow \{a, b\}^*$$

$$(L^*)^* \Rightarrow (\{a, b\}^*)^* = \{a, b\}^* \\ = L^* \quad \forall \Sigma$$

⑩ (a) $(L \cup L_2)^R = L^R \cup L_2^R \quad \forall L, L_2$

$$(L \cup L_2)^R \Rightarrow \text{if } u \in L, v \in L_2$$

$$uv \in L \cup L_2,$$

$$(L \cup L_2)^R = (uv)^R = v^R u^R \\ \neq L_1^R \cup L_2^R$$

(EXERCISES)

B. (b)

$$(L^R)^* = (L^*)^R \neq L$$

let $uv \in L$

$$\begin{aligned} L^R &= (uv)^R \\ &= v^R u^R \end{aligned}$$

$$(L^R)^* = (v^R u^R)^*$$

$$L^* = (uv)^*$$

$$(L^*)^R = [(uv)^*]^R$$

$$\therefore (L^R)^* \neq (L^*)^R$$

11 Find the Grammars that generate the sets of following for $\Sigma = \{a, b\}$

(a) all strings with exactly one a.

P:

$$\begin{aligned} S &\rightarrow AaA \\ A &\rightarrow bA/\lambda \end{aligned}$$

$$G = (\{A, S\}, \{a, b\}, S, P)$$

(b) all strings with atleast one a.

P:

$$\begin{aligned} S &\rightarrow AaA \\ A &\rightarrow aA/bA/\lambda \end{aligned}$$

$$G = (\{A, S\}, \Sigma, S, P)$$

(c) all strings with no more than 3 a's.

P:

$$\begin{aligned} S &\rightarrow AaAaAaA \quad 0, 1, 2, 3 \text{ a's} \\ A &\rightarrow bA/\lambda \end{aligned}$$

$$\begin{aligned} S &\rightarrow AaA/\lambda AaAaA/\lambda \\ &\quad AaAaAaA \end{aligned}$$

$$G = (\{A, S\}, \Sigma, S, P) \quad A \rightarrow bA/\lambda$$

(d) all strings with atleast 3 a's.

P:

$$\begin{aligned} S &\rightarrow AaAaAaA \\ A &\rightarrow aA/bA/\lambda \end{aligned}$$

$$G = (\{A, S\}, \Sigma, S, P)$$

Test:

$$aaa: \quad S \rightarrow AaAaAaA \rightarrow aaa$$

$$baababa: \quad S \rightarrow AaAaAaA \rightarrow baAaAaA \rightarrow$$

$$baaAaAaA \rightarrow baabaAaA \rightarrow$$

$$baababa \checkmark$$

$S \rightarrow AaA/\lambda AaAaA/\lambda$
 $A \rightarrow bA/\lambda$

(12)

$$S \rightarrow aA$$

$$A \rightarrow bS$$

$$S \rightarrow \lambda$$

ab, abab, ...

$$L = \{(ab)^n : n \geq 0\}$$

(13)

What language does the Grammar with these productions generate?

$$S \rightarrow Aa$$

$$A \rightarrow B$$

$$B \rightarrow Aa$$

$$S \rightarrow Aa \rightarrow Ba \rightarrow Aaa$$

$L = \emptyset$ \because no terminal symbol to generate strings.

(14)

$\Sigma = \{a, b\}$. For each of below languages, find a grammar that generates it

(a) $L_1 = \{a^n b^m : n \geq 0, m \geq n\}$

P_1 :

$$S \rightarrow Ab$$

$$A \rightarrow aAb / \lambda / Ab$$

$$G : (\{A, S\}, \{a, b\}, S, P_1)$$

test: $b : S \rightarrow Ab \rightarrow b \checkmark$

$$abb \rightarrow Ab \rightarrow aAbb \rightarrow abb$$

ab: $S \rightarrow Ab \times$

$$bb \rightarrow S \rightarrow Ab \rightarrow bb \checkmark$$

$S \rightarrow aSb / B$
 $B \rightarrow bB / b$

(b)

$$L_2 = \{a^n b^{2n} : n \geq 0\}$$

P_2 :

$$S \rightarrow aSbb / \lambda$$

$$G : (\{S\}, \Sigma, S, P_2)$$

test:

$$\lambda : S \rightarrow \lambda$$

$$aab : S \rightarrow aSbb \times$$

$$abb : S \rightarrow aSbb \rightarrow abb \checkmark$$

(c)

$$L_3 = \{a^{n+2} b^n : n \geq 1\}$$

$$S \rightarrow aaA$$

$$A \rightarrow aAb / \lambda$$

test:

$$n=1 : a^3 b : S \rightarrow aaA \rightarrow aaAaB \rightarrow aaab$$

14.

(c)

$$L_2 = \{a^n b^{n-3} : n \geq 3\}$$

$$\Rightarrow L_3 = \{a^{m+3} b^m : m \geq 0\}$$

 $P_3:$

$$S \rightarrow aaaA$$

$$A \rightarrow aSb / \lambda$$

$$\therefore G = (\{A, S\}, \Sigma, S, P_3)$$

$$n-3 = m$$

$$n = m+3$$

$$n \geq 3$$

$$m \geq 0$$

(d)

$$L_3 = L_1 L_2$$

$$L_5 = \{a^n b^m a^n b^{2n} : n \geq 0, m \geq n\}$$

$$= \{a^n b^m a^k b^{2k} : n, k \geq 0, m \geq n\}$$

$$S \rightarrow AB$$

$$A \rightarrow Cb$$

$$C \rightarrow aCb / \lambda / Cb$$

$$B \rightarrow aBbb / \lambda$$

 \Rightarrow

$$S \rightarrow AbB$$

$$A \rightarrow aAb / \lambda / Ab$$

$$B \rightarrow aBbb / \lambda$$

$$S \rightarrow S_1 S_2$$

$$\underline{\text{tst:}} \quad abbabb: \quad S \rightarrow AbB \rightarrow aAbbB \rightarrow abbB \rightarrow abbabb \checkmark$$

$$b: \quad S \rightarrow AbB \rightarrow bB \rightarrow b \checkmark$$

$$bb: \quad \times$$

(e) $L_1 \cup L_2:$

$$S \rightarrow Ab / B$$

$$A \rightarrow aAb / \lambda$$

$$B \rightarrow aBbb / \lambda$$

$$S \rightarrow S_1 / S_2$$

(g)

$$L_1^3: \{a^n b^m a^n b^m a^n b^m : n \geq 0, m > n\}$$

$$S \rightarrow AbAbAb$$

$$A \rightarrow aAb / Ab / \lambda$$

$$S \rightarrow SS_1$$

Test: $n=0, m=1$ bbb

$$S \rightarrow AbAbAb \rightarrow bbb$$

reject: abababa

$$S \rightarrow AbAbAb$$

$$\rightarrow aAbbAbAb \quad \times$$

$n=0, m=2$ bbbbbb

$$S \rightarrow AbAbAb \rightarrow bbbbbb$$

(h)

 L_1^*

$$L_1 = \{a^n b^m : n \geq 0, m > n\}$$

$$S \rightarrow SA / \lambda$$

$$A \rightarrow aAb / Ab / \lambda$$

$$S \rightarrow SS_1 / \lambda$$

test: $\lambda: S \rightarrow \lambda$

abbaabbb: $S \rightarrow SA \rightarrow SaAb \rightarrow Sa aAbb \rightarrow Sa abbb \rightarrow$
 $SA aabbb \rightarrow aAbaabbb \rightarrow abbaabbb \quad \checkmark$

reject:

aba: $S \rightarrow SA \rightarrow SaAb \quad \times$

(i)

 $L_1 - \overline{L_4}$

$$L_4 = \{a^{n+3} b^m : n \geq 0\}$$

$$L_1 = \{a^n b^m : n \geq 0, m > n\}$$

$$L_1 - \overline{L_4} = L_1 - (U - L_4)$$

$$= L_1 - U + L_4$$

$$= L_1 + L_4 - U = \emptyset$$

(15)

Find the grammars for the following on $\Sigma = \{a\}$

(a) $L = \{w : |w| \bmod 3 = 0\}$

$S \rightarrow aaaS / \lambda$

(b) $L = \{w : |w| \bmod 3 > 0\}$

1, 2, 4, 5, 7, 8, 10, 11

$S \rightarrow A / B$

$A \rightarrow aaaA / a$

$S \rightarrow aaaS / a / aa$

$B \rightarrow aaAB / aa$

Test:

$aaaaaaaa: S \rightarrow A \rightarrow aaaA \rightarrow aaaaaaaA \rightarrow aaaaaaaaa$

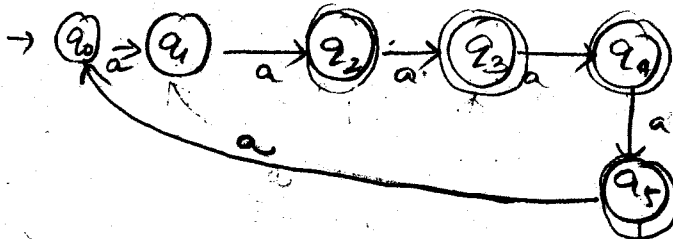
$aaaaaaaaaa: \times$

(c) $L = \{w : |w| \bmod 3 \neq |w| \bmod 2\}$

$\bmod 3$
 \downarrow
 $\{0, 1, 2\}$

$\bmod 2$
 \downarrow
 $\{0, 1\}$

\rightarrow skip every 6th & 7th



$S \rightarrow aaA$

$S \rightarrow aa/aaa/aaaa/aaaaa/aaaaaa$

$A \rightarrow \lambda/a/aa/aaa/aaaaS/aaaaaaA$

Test:

$\lambda \times$
 $a \times$
 $aa \checkmark$

$9a's \checkmark$

$8a's \checkmark$

$13a's \times$

$S \rightarrow aaA$

$A \rightarrow \lambda/a/aa/aaa/aaaaS$

#	mod 2	mod 3
0	0	0
1	1	1
2	0	2
3	1	0
4	0	1
5	1	2
6	0	0
7	1	1
8	0	2
9	1	0
10	0	1
11	1	2
12	0	0
13	1	1
14	0	2

(d) $L = \{w: |w| \bmod 3 \geq |w| \bmod 2\}$

$ w $	$\bmod 2$	$\bmod 3$	\geq
0	0	0	✓
1	1	1	✓
2	0	2	✓
3	1	0	✗
4	0	1	✓
5	1	2	✓
6	0	0	✓
7	1	1	✓
8	0	2	✓
9	1	0	✗
10	0	1	✓
11	1	2	✓
12	0	0	✓
13	1	1	✓
14	0	2	✓
15	1	0	✗

$$S \rightarrow \lambda / a / aa / aaa A$$

$$A \rightarrow a / aa / aaa / aaaa / aaaaa / aaaaaa A$$

Test: 3a's: $S \rightarrow aaa A$ ✗

5a's: $S \rightarrow aaaa \rightarrow aaaaa$ ✓

(16)

Find a grammar that generates the language $L = \{ww^R: w \in \{a,b\}^+\}$

$$S \rightarrow aSa / bSb / ab / ba \quad \{a,b\}^+$$

abba: $S \rightarrow aSa \rightarrow abSba \rightarrow abba$

bbbb: $S \rightarrow bSb \rightarrow bbSbb \rightarrow bbbb$

$$S \rightarrow aSa / bSb / ab / ba$$

(17)

Give verbal description of

$$S \rightarrow aSb / bSa / a$$

In no order of a and b, no. of a's are ^{one} more in any string.

$$S \rightarrow aSb \rightarrow aaSbb \rightarrow aaabb$$

$$\rightarrow bSa \rightarrow baa$$

$$bSa \rightarrow bbsaa \rightarrow bbaaa$$

b's: n

a's: n+1

(a) $L = \{w : n_a(w) = n_b(w) + 1\}$

we know for $L = \{w : n_a(w) = n_b(w)\}$ equal a's & b's

$G \Rightarrow S \rightarrow SS$

$S \rightarrow asb / bSa / \lambda$

$A \rightarrow AA / aAb / bAa / \lambda$

$S \rightarrow \boxed{AaA}$

additional A

$\therefore S \rightarrow SSa / aSS / asb / bSa / (\lambda) / SaS$

Test:

$S \rightarrow SSa \rightarrow asbSa \rightarrow a\lambda b\lambda a \rightarrow aba \rightarrow \boxed{aba} \checkmark$

$S \rightarrow \boxed{a} \checkmark$

$S \rightarrow SSa \rightarrow asbbSa \rightarrow a\lambda bbb\lambda a \rightarrow \boxed{abbaa} \checkmark$

$S \rightarrow SSa \rightarrow bSa \rightarrow bbsaaaa \rightarrow \boxed{bbbaaaa} \checkmark$

$S \rightarrow SaS \rightarrow bSaasb \rightarrow \boxed{baaab} \checkmark$

(b) $L = \{w : n_a(w) > n_b(w)\}$ $S \rightarrow SS / asb / bSa / aS / a$ add any no. of a's

$S \rightarrow \cancel{SS / aS} / asb / bSa / (\lambda)$

(c) $L = \{w : n_a(w) = 2n_b(w)\}$

$S \rightarrow SS / aSba / aaSb / bSaa / abSa / aSab / baSa / \lambda$

test: reject $aabb$: $aasb \rightarrow aab \times$

$aaab$: $aasb \rightarrow aa \times$

aab : $S \rightarrow aab \checkmark$

$ababbbaaa$: $S \rightarrow SS \rightarrow abSa \rightarrow ababSaa \rightarrow ababbbaSaaa$

$aaaaaaabbb$: $SS \rightarrow aasb \rightarrow aaaaaSbb$

$\rightarrow aaaaaaasbbb \rightarrow aaaaaaabbb \checkmark$

(d) $L = \{w \in \{a,b\}^* : |n_a(w) - n_b(w)| = 1\}$ Equal as p^b

$\Rightarrow n_a(w) = n_b(w) + 1$ ✓
 $n_b(w) = n_a(w) + 1$ ✓

$A \rightarrow AA/aAb/bAa/\lambda$

$S \rightarrow AaA/ABa$

$S \rightarrow A/B$

$A \rightarrow AA/aAA/AaA/aAb/bAa/\lambda$

$B \rightarrow BBb/bBB/BbB/aBb/bBa/\lambda$

(19)

$\Sigma = \{a,b,c\}$

(a) $L = \{w : n_a(w) = n_b(w) + 1\}$

we know for

$\Sigma = \{a,b\}$

$n_a(w) = n_b(w)$

$S \rightarrow SS/aSb/bSa/\lambda$

$a=b : c \text{ varying}$

$S \rightarrow SS/aSb/bSa/cS/\lambda$

$a=b+1 : c \text{ varying}$

$S \rightarrow AaA$

$A \rightarrow AA/aAb/bAa/cA/\lambda$

$\therefore \Sigma = \{a,b,c\} : S \rightarrow SS/aSb/bSa/C$

$C \rightarrow cC/\lambda$

~~$S \rightarrow AaA$
 $A \rightarrow aAb/bAa/c$
 $C \rightarrow cC/\lambda$~~

$\therefore n_a(w) = n_b(w) + 1 \Rightarrow$

$S \rightarrow aSS/SSa/saS/aSb/bSa/C$

$C \rightarrow cC/\lambda$

(or) $S \rightarrow SSa/aSS/saS/aSb/bSa/cS/\lambda$

(b) $L = \{w : n_a(w) \geq n_b(w)\}$

$S \rightarrow SS/aSb/bSa/CS/aS/a$

$S \rightarrow SS/aS/aSb/bSa/cS/\lambda$

(c) $L = \{w : n_a(w) = 2n_b(w)\}$

$S \rightarrow SS/cS/aSb/asba/aSab/abSa/baSa/bSaa/\lambda$

(d) $L = \{w : |n_a(w) - n_b(w)| = 1\}$

$S \rightarrow S_1 S_2$ add one a/
 add one b

$S \rightarrow AaA/AbA$

$A \rightarrow aAb/bAa/AA$

$S_2 \rightarrow S_2 S_2 / cS_2 / baSb/abSb/bbSa/bSab/bSba/\lambda cA/\lambda$

PT.

$$S \rightarrow aAb / \lambda$$

$$A \rightarrow aAb / \lambda$$

generates $\{a^n b^n : n \geq 0\}$

$$S \rightarrow \lambda$$

$$S \rightarrow aAb \rightarrow ab$$

$$S \rightarrow aAb \rightarrow aaAbb \rightarrow aabb$$

$$\therefore L = \{\lambda, ab, aabb, \dots\}$$

$$L = \{a^n b^n : n \geq 0\} \text{ is true}$$

(21)

$$S \rightarrow aSb / ab / \lambda$$

\equiv

$$S \rightarrow aAb / ab$$

$$A \rightarrow aAb / \lambda$$

$$S \rightarrow \lambda$$

$$S \rightarrow ab$$

$$S \rightarrow aSb \rightarrow aabb$$

$$L = \{a^n b^n : n \geq 0\}$$

$$S \rightarrow ab$$

$$S \rightarrow aAb \rightarrow ab$$

$$S \rightarrow aaAbb \rightarrow aabb$$

$$L = \{a^n b^n : n > 0\}$$

as both grammars represent different languages,
they are not equivalent.

(22)

ST. $S \rightarrow SS / SSS / aSb / bSa / \lambda$ is equivalent to $S \rightarrow SS / aSb / bSa / \lambda$.

If we rewrite SS as SSS $\{ \because S \rightarrow SS \}$

both are representing same Grammars.

$$\text{where } n_a(w) = n_b(w)$$

(23)

$$\text{ST. } S \rightarrow aSb / bSa / SS / a$$

\neq

$$S \rightarrow aSb / bSa / a$$

$$S \rightarrow a$$

$$S \rightarrow SS \rightarrow aa$$

$$aa \in L_1$$

$$S \rightarrow aSb \rightarrow aab$$

$$aa \notin L_2$$

CHAPTER 1-3

Example
1.15

- ① id is a sequence of letters, digits, underscores
- ② id must start with a letter or underscore
- ③ id allow upper & lower case letters.

$\langle id \rangle \rightarrow \langle letter \rangle \langle rest \rangle / \langle undscr \rangle \langle rest \rangle$

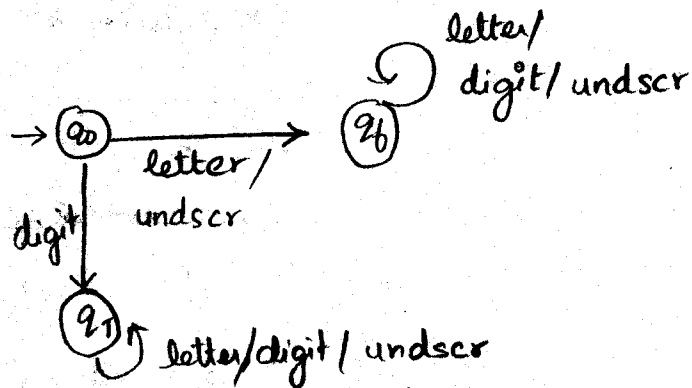
$\langle rest \rangle \rightarrow \langle letter \rangle \langle rest \rangle / \langle digit \rangle \langle rest \rangle / \langle undscr \rangle \langle rest \rangle / \lambda$

$\langle letter \rangle \rightarrow a|b|c| \dots |z|A|B|C| \dots |Z$

$\langle digit \rangle \rightarrow 0|1|2| \dots |9$

$\langle undscr \rangle \rightarrow -$

1.16



Example
1.17

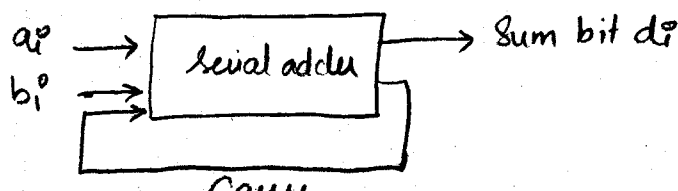
Binary Adder

		b_i	
		0	1
a_i	0	No Carry	No Carry
	1	No Carry	Carry

$$V(x) = \sum_{i=0}^n a_i 2^i$$

$$i/p : (a_i, b_i)$$

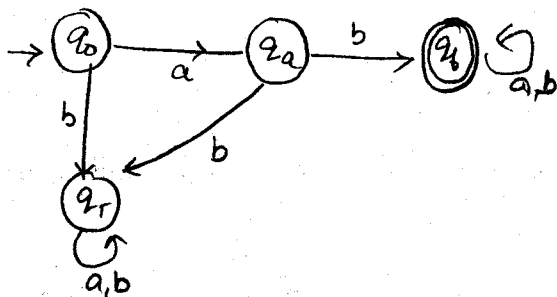
$$o/p = \text{sum bit } d_i$$



Example

2.3

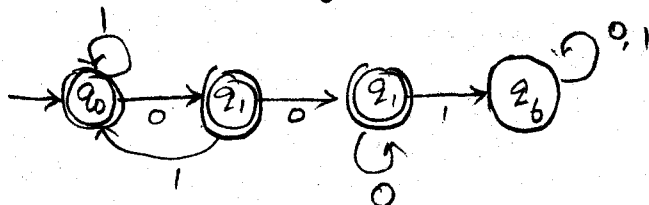
Find a dfa that recognises all strings on $\Sigma = \{a, b\}$ with prefix ab .



Example

2.4

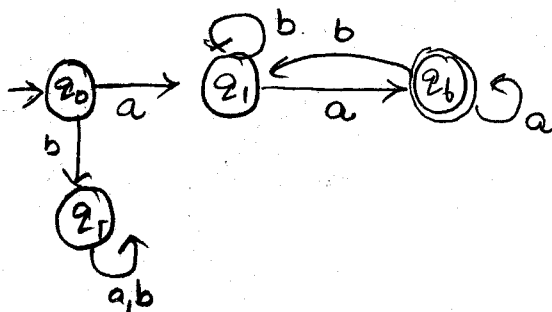
Find a dfa that accepts all the strings on $\{0,1\}$ except those containing the substring 001 .



Example

2.5

Show that $L = \{awa : w \in \{a,b\}^*\}$ is regular.



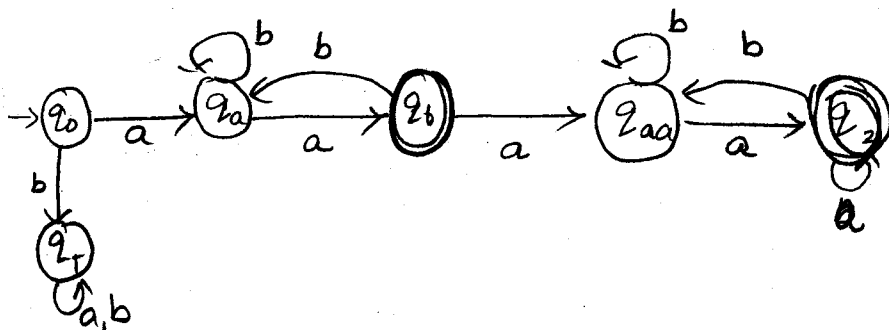
Show that Regular \Rightarrow
Draw Dfa

Example

2.6

$L = \{aw_1aw_2a : w_1, w_2 \in \{a,b\}^*\}$ is regular.

L regular $\Rightarrow L^1, L^2, L^3, \dots$ are also regular.

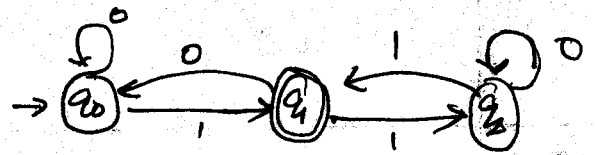


EXERCISES

① Which of following are accepted by

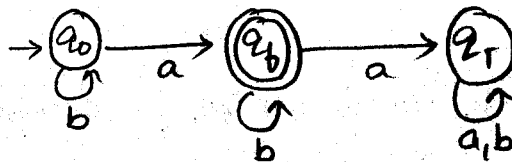
0001 ✓
01001 ✓

0000110. X

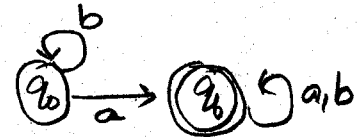


② for $\Sigma = \{a, b\}$ construct dfa's

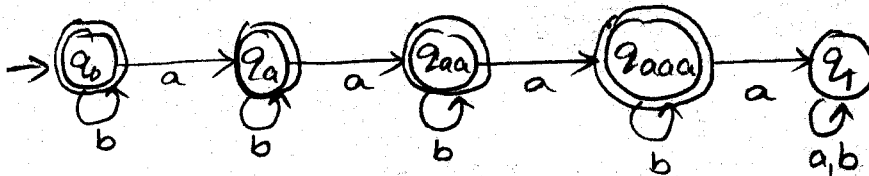
(a) all strings with exactly one a.



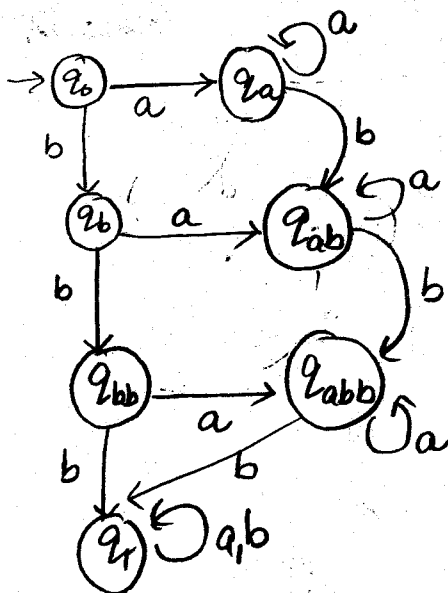
(b) all strings with @least one a



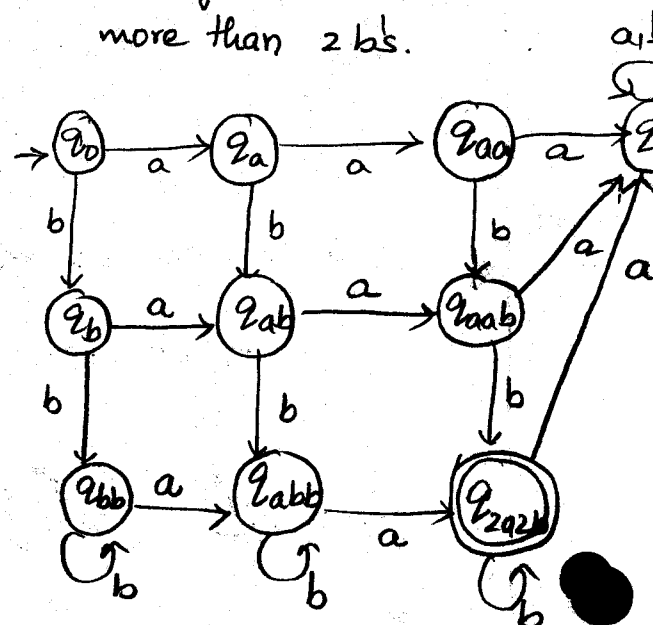
(c) all strings with no more than 3 a's.



(d) @least one a & exactly 2 b's.

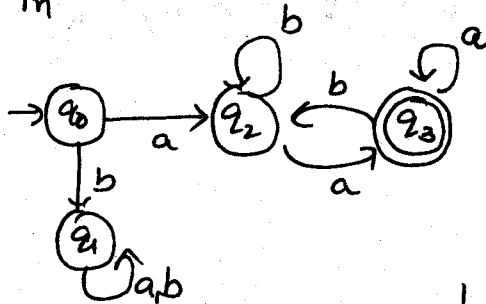


(e) all strings with 2 a's & more than 2 b's.



③

ST in

If: $q_3 \notin F$ $q_0, q_1, q_2 \in F$, resulting
dfa accepts \bar{L} .

$$L = \{awa : w \in \Sigma^*\}$$

 \bar{L} : is accepted by the changes to L .

④

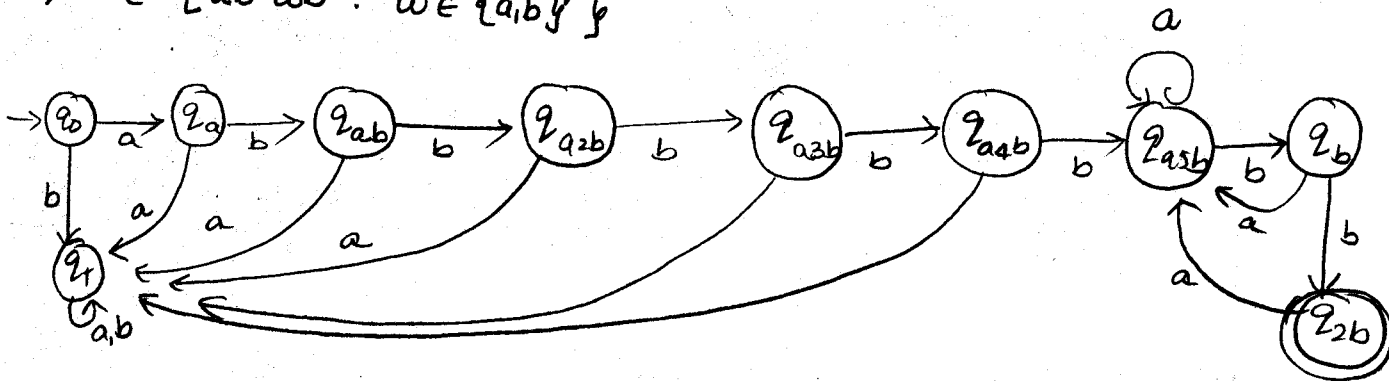
$$M = (Q, \Sigma, \delta, q_0, F)$$

$$\hat{M} = (Q, \Sigma, \delta, q_0, Q-F)$$

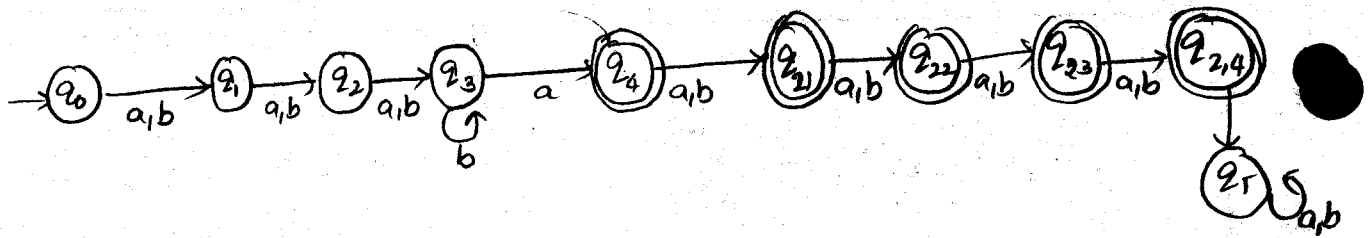
$$\text{then } \overline{L(M)} = L(\hat{M})$$

⑤

$$(a) L = \{ab^5wb^2 : w \in \{a,b\}^*\}$$

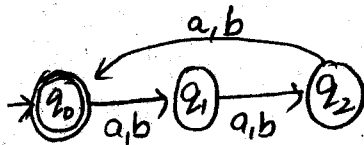


⑥ $\Sigma = \{a, b\}$ give dfa for $L = \{w_1 a w_2 : |w_1| \geq 3, |w_2| \leq 5\}$

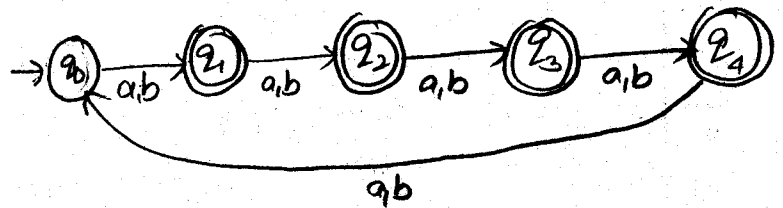


⑦ on $\Sigma = \{a, b\}$

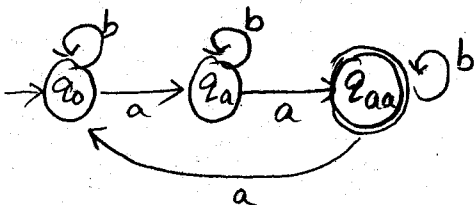
(a) $L = \{w : |w| \bmod 3 = 0\}$



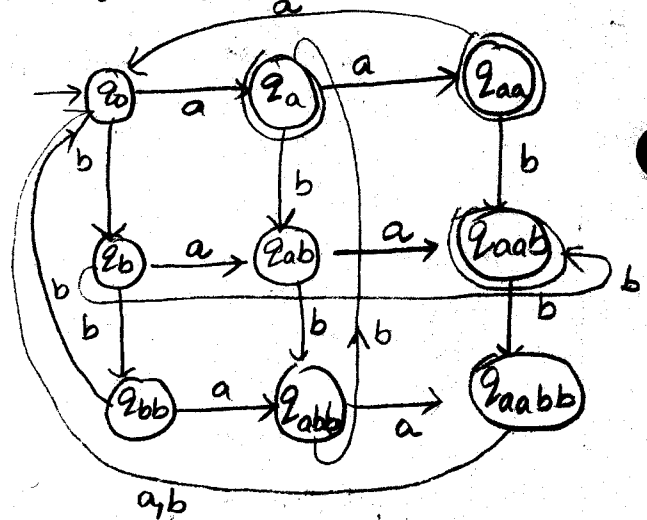
(b) $L = \{w : |w| \bmod 5 \neq 0\}$



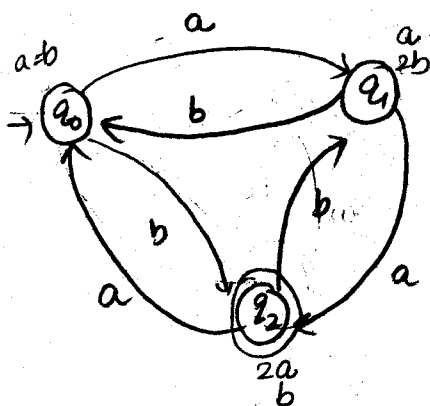
(c) $L = \{w : n_a(w) \bmod 3 > 1\}$



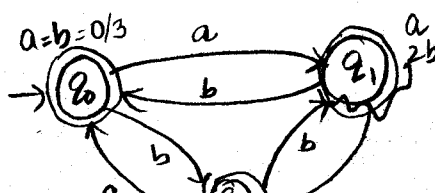
(d) $L = \{w : n_a(w) \bmod 3 > n_b(w) \bmod 3\}$



(e) $L = \{w : (n_a(w) - n_b(w)) \bmod 3 > 0\}$



(f) $L = \{w : (n_a(w) + 2n_b(w)) \bmod 3 < 2\}$



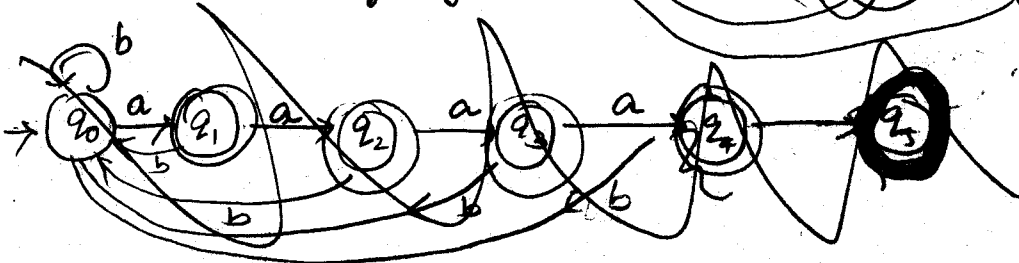
8

run = substring of length at least 2
 & entirely of same symbol

on $\Sigma = \{a, b\}$ find dfa for

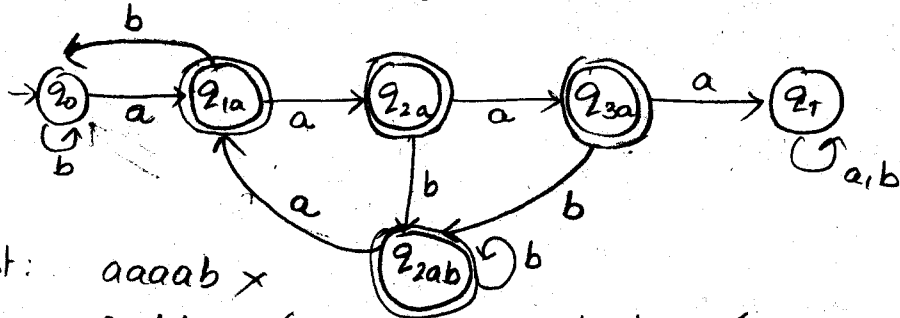
(a)

$L = \{w : \text{no runs of length } < 4\}$



(b)

$L = \{w : \text{every run of a's has length 2 or 3}\}$



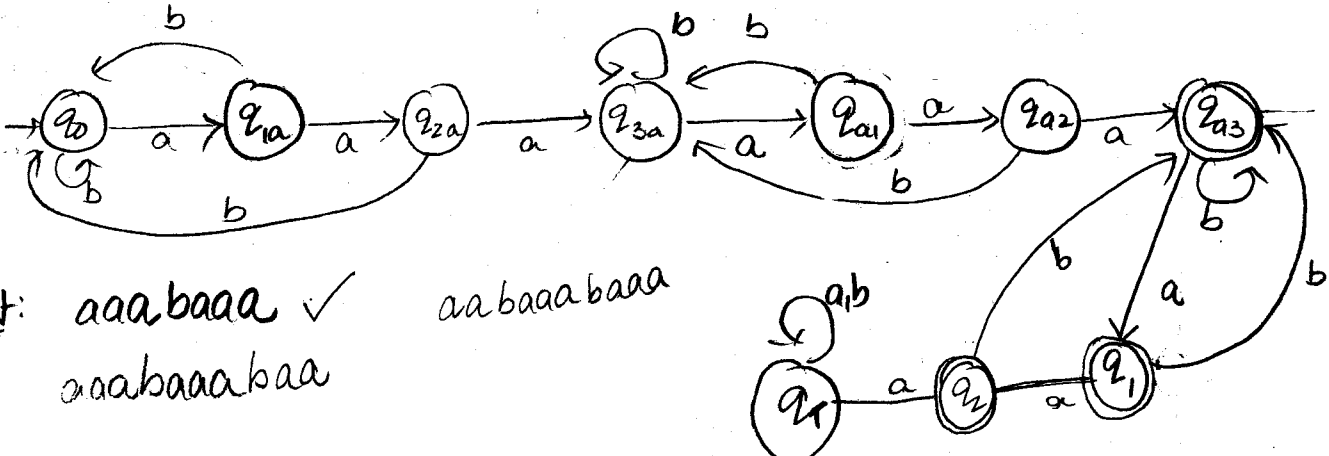
Test: aaaab x
 aabba ✓
 baaab ✓

baba ✓
 baaaa x

{ zero, one a should be accepted also }

(c)

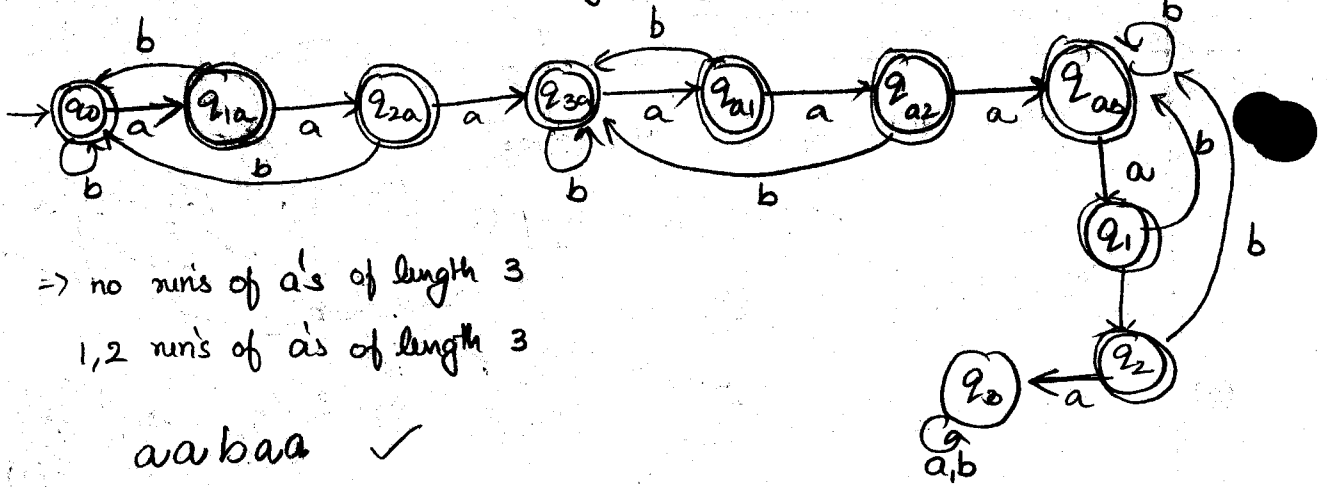
$L = \{w : \text{exactly 2 runs of a's of length 3}\}$
 waaaaw / waaaawaaaaw



Test: aaabaaa ✓
 aaabaaaabaa

aaabaaaabaaa

(d) $L = \{w: \text{@most 2 runs of a's of length 3}\}$

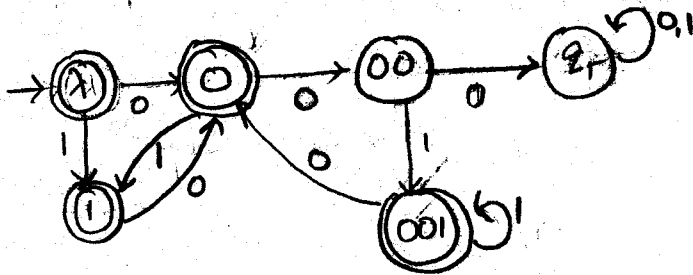


\Rightarrow no runs of a's of length 3
1, 2 runs of a's of length 3

aaabaa ✓
aaab
aaabaaa

9 $\Sigma = \{0, 1\}$

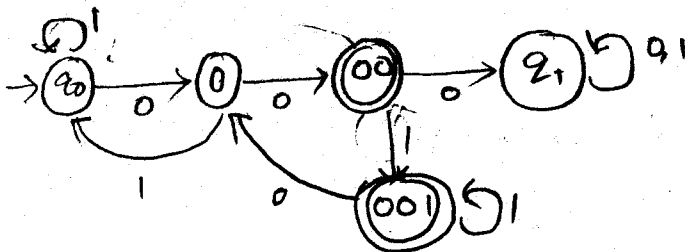
(a) Every 00 followed by 1



Test:

00100111 ✓
010001 X
01 ✓
10 ✓
11 ✓
01 ✓

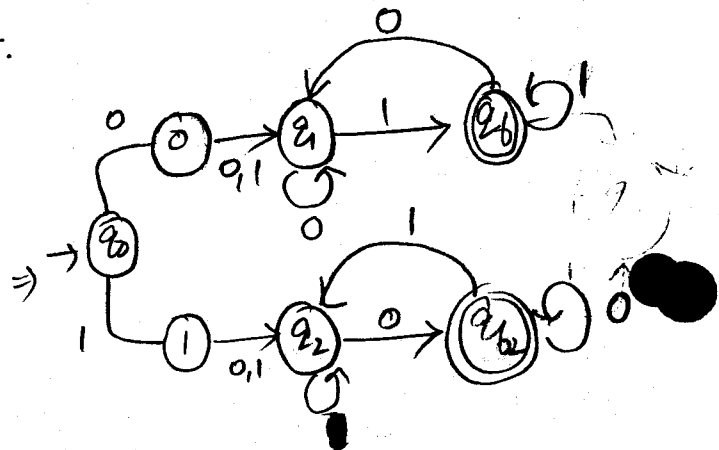
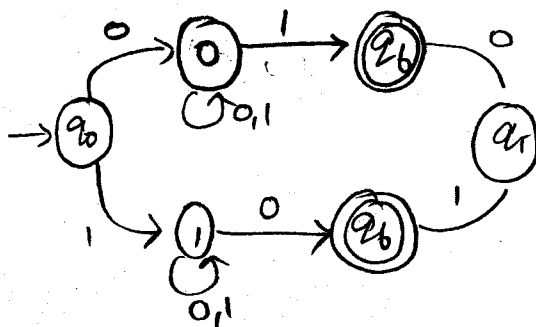
(b) All strings containing 00 but not 000.



Test: 0010011 ✓

00011 ✓
1001000 X

(c) leftmost differs from rightmost.



1 WD

nba

Nfa:

$$M = (Q, \Sigma, \delta, q_0, F)$$

$$\delta: Q \times (\Sigma \cup \{\lambda\}) \rightarrow 2^Q$$

Example
2.7

fig 2.8

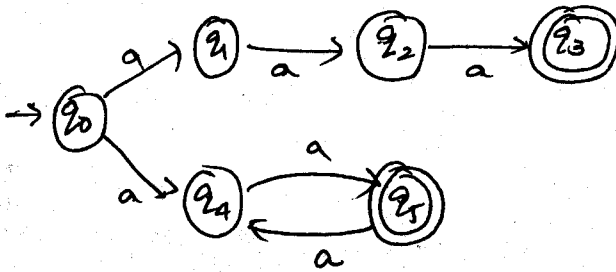
Example
2.8

fig 2.9

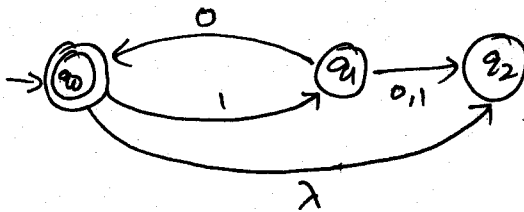
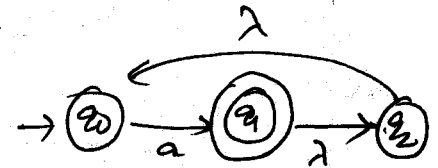
Example
2.9

fig 2.10

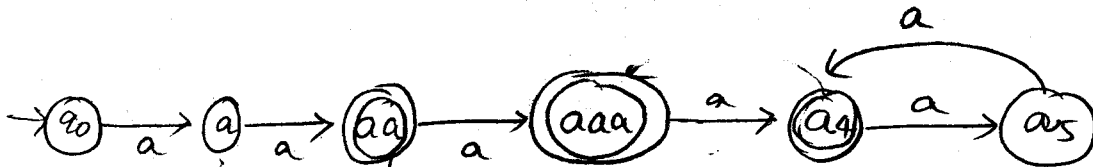


$$L(M) = \{w \in \Sigma^* : \delta^*(q_0, w) \cap F \neq \emptyset\}$$

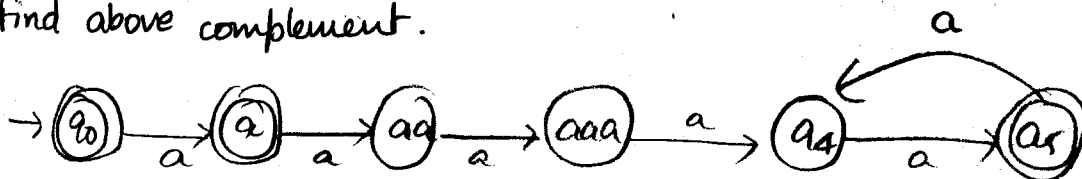
EXERCISES

② find dfa defined by: fig 2.8

$$L: \{aaa\} \cup \{a^{2n} : n \geq 1\}$$



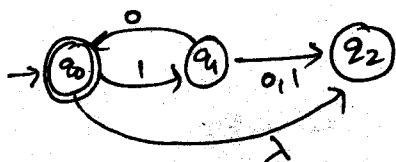
③ find above complement.



④ Hg^{2+} $\delta(2_0, 1011)$: $\delta(2_1, 1011)$ / $\delta(2_0, 11)$ \rightarrow ϵ_2
 $\delta^*(2_1, 01)$ ϵ_2'

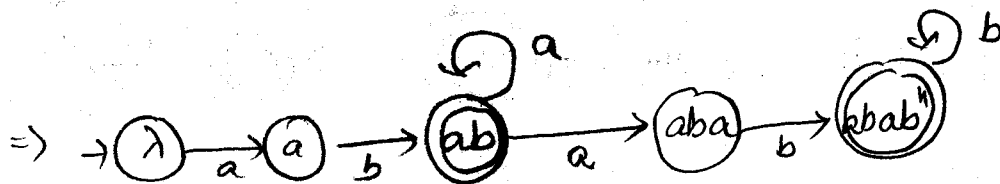
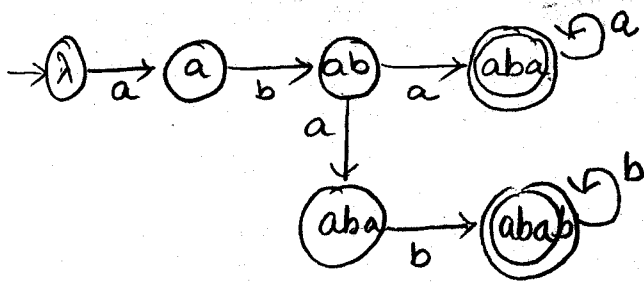
⑤ fig 2.10: $8^*(q_0, a)$ $\{q_0, q_1, q_2\}$
 $8^*(q_1, \lambda)$ $\{q_0, q_2\}$

⑥ for:

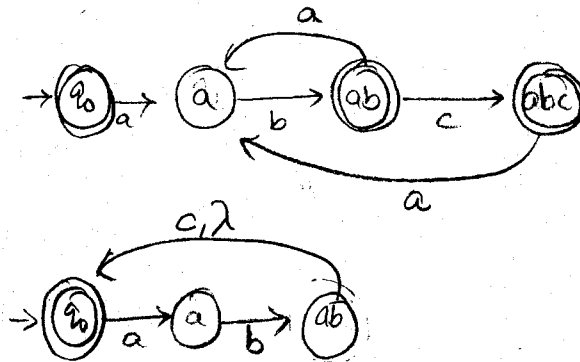


$$\begin{aligned}
 & \delta^*(q_0, 1010) \rightarrow \delta^*(q_1, 010) \rightarrow \delta^*(q_0, 10) \rightarrow \delta^*(q_1, 0) \rightarrow q_2 \\
 & \delta^*(q_1, 00) \rightarrow \delta^*(q_2, 1010) \rightarrow \delta^*(q_2, 10) \quad \times \\
 & \delta^*(q_1, 00) \rightarrow \delta^*(q_0, 0) \rightarrow q_2 \\
 & \delta^*(q_2, 0) \quad \times
 \end{aligned}$$

⑦ no more than 5 states, design nfa for $\{abab^n : n \geq 0\} \cup \{aba^n : n \geq 0\}$



8 Construct nfa with 3 states for $\{ab, abc\}^*$



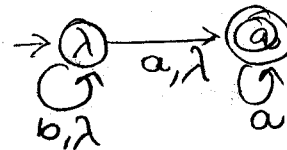
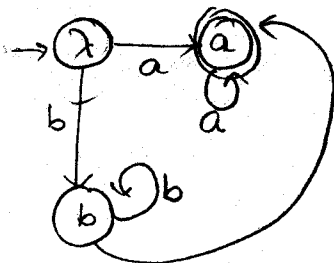
9 Can it be done in fewer states than 3?

No as $|abab^n|$ least = 3 for $n=0$.

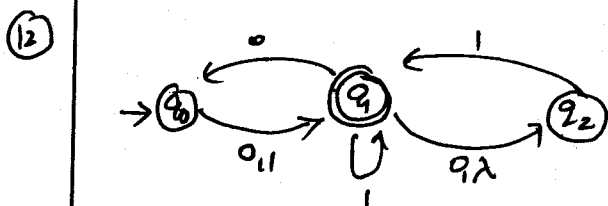
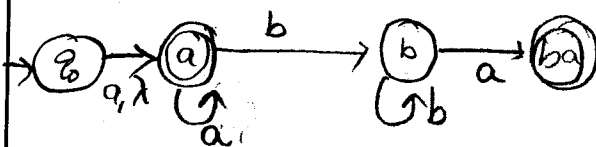
10 find nfa with 3 states that accepts

$L = \{a^n : n \geq 1\} \cup \{b^m a^k : m \geq 0, k \geq 0\}$

(b) can fewer than 3 states be possible?



11 Nfa - 4 states for $L = \{a^n : n \geq 0\} \cup \{b^n a : n \geq 1\}$



00 : $\delta^*(q_0, 00) \rightarrow \{q_0, q_2\} \cap F = \emptyset$ reject

01001 : $\{q_1\} \cap F \neq \emptyset$ accept

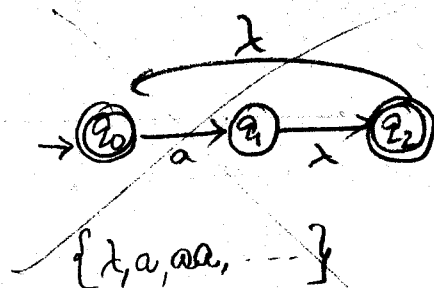
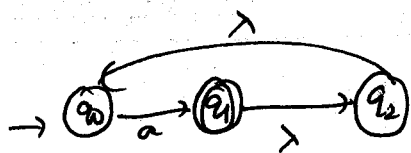
10010 : $\{q_0, q_2\} \cap F = \emptyset$ reject

000 : $\{q_1, q_2\} \cap F \neq \emptyset$ accept

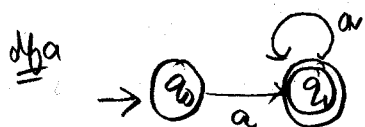
0000 : $\{q_0, q_2\} \cap F = \emptyset$ reject

13

What is complement of

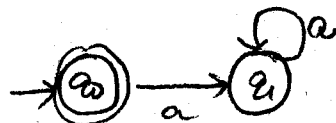


can't take complement of nfa



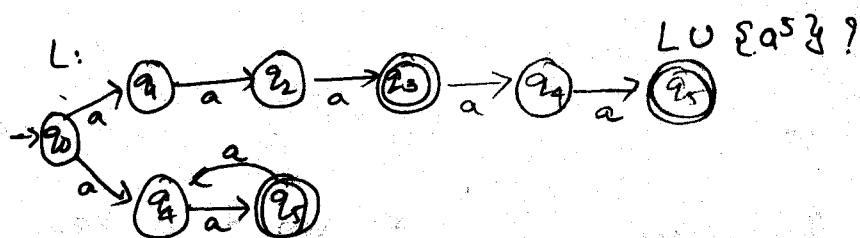
all strings $n_a(w) > 1$

complement



all strings $n_a(w) < 1$

14

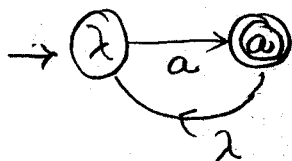


15

is? $\{a\}^* - \{a\}^+$

16

Find nfa for $\{a\}^*$ such that removing one edge will accept $\{a\}$



17

Can above be done with dfa?

No: need two paths to accept one a / more a's

18

let $p_0 \in Q_0$.

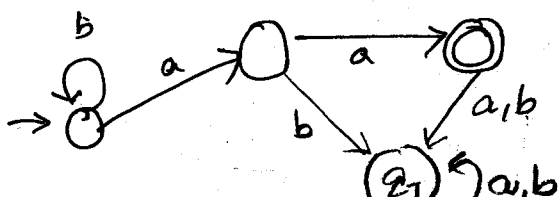
$\delta(p_0, \lambda) \rightarrow Q_0$

if Q_0 not initial, equivalent to M .

19

Yes.

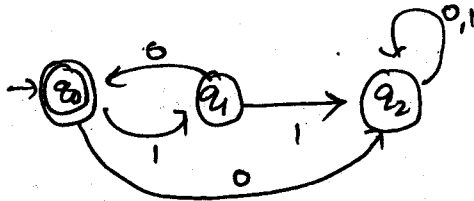
20



Equivalence of Nfa & Dfa:

fig 2.11

dfa

 $\{\lambda, 10, 1010, \dots\}$ $\Rightarrow \{(10)^n : n \geq 0\}$

nfa

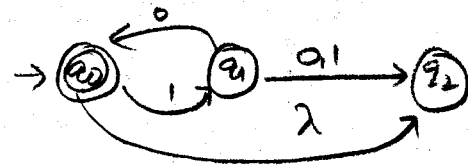
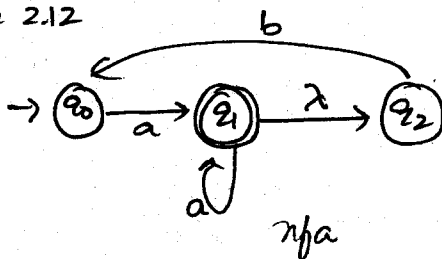
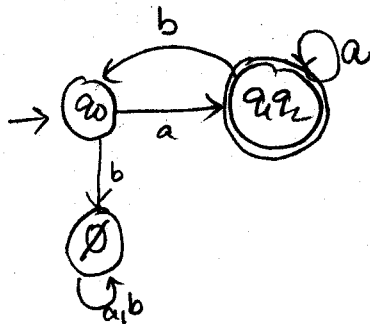
 $\{(10)^n : n \geq 0\}$

fig 2.12

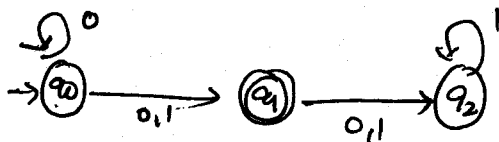


nfa

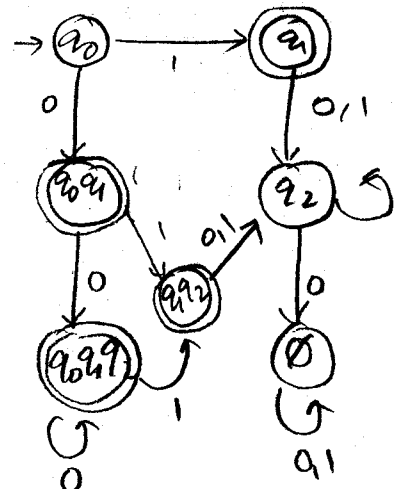
dfa =



	a	b
$\{q_0\}$	$\{q_1, q_2\}$	\emptyset
$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_0\}$
\emptyset	\emptyset	\emptyset

Example
2.13

	0	1
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_1\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$
$\{q_1\}$	$\{q_2\}$	$\{q_2\}$
$\{q_2\}$	\emptyset	$\{q_2\}$
$\{q_1, q_2\}$	$\{q_2\}$	$\{q_2\}$

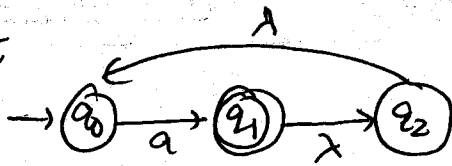


If too many
states getting
combined,
don't end,
go on &
enumerate
all

2-3
EXERCISES

①

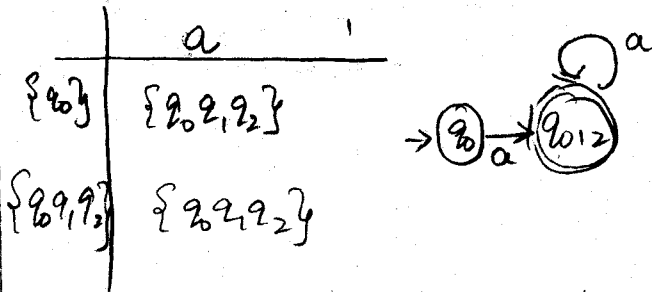
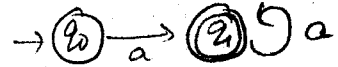
Convert



to dfa. Is there a simpler way?

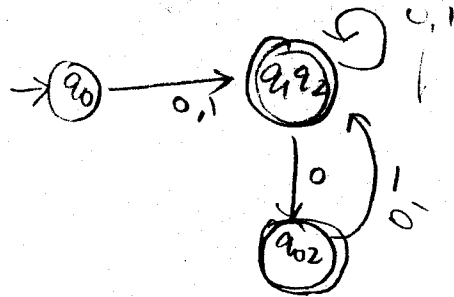
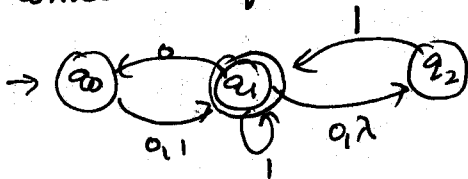
Yes:
directly.

$$L = \{a^n : n > 0\}$$



②

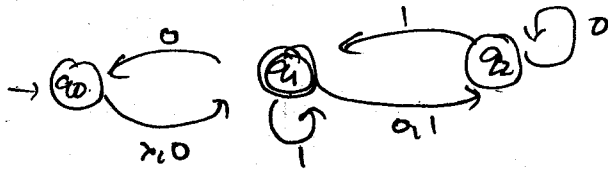
Convert to dfa



	0	1
{q0}	{q1, q2}	{q1, q2}
{q1, q2}	{q0, q2}	{q1, q2}
{q0, q2}	{q1, q2}	{q1, q2}

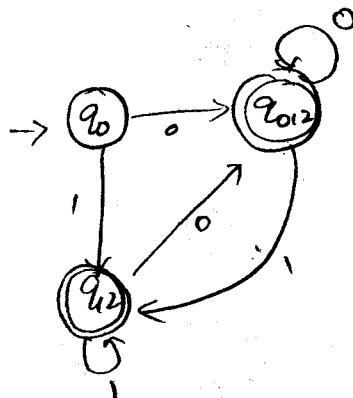
③

Convert nfa → dfa



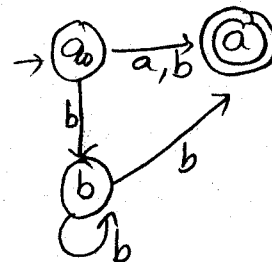
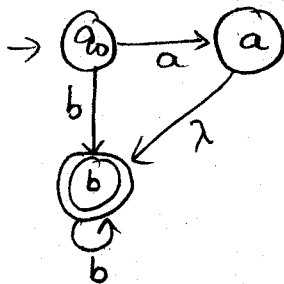
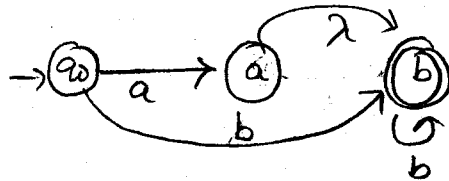
$$q_0 = q_1$$

	0	1
{q0}	{q0, q1, q2}	{q1, q2}
{q1, q2}	{q0, q1, q2}	{q1, q2}
{q0, q1, q2}	{q0, q1, q2}	{q1, q2}



⑧ Find nfa without λ -transitions, single final state for

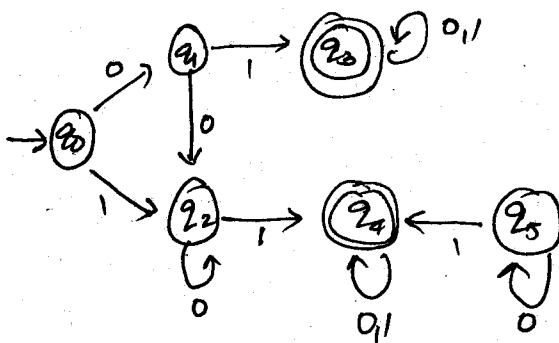
$$\{a\} \cup \{b^n : n \geq 1\}$$



CH # 2.4

(Reduction of states in dfa)

Example
 2.17



	q_0	q_1	q_2	q_3	q_4
q_0		D	D	D	D
q_1			ID	D	D
q_2				D	D
q_3					ID
q_4					

States: q_5

D: $\{q_0, q_1\}$
 $\{q_0, q_2\}$

ID: $\{q_3, q_4\}$
 $\{q_1, q_2\}$

$\{q_0\}, \{q_1, q_2\}, \{q_3, q_4\}$

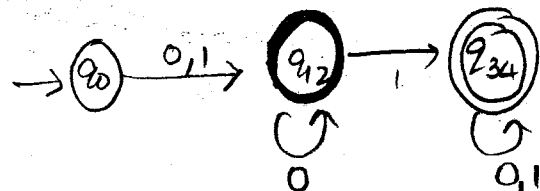
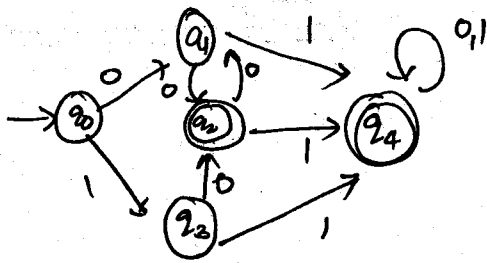


fig: 2.18

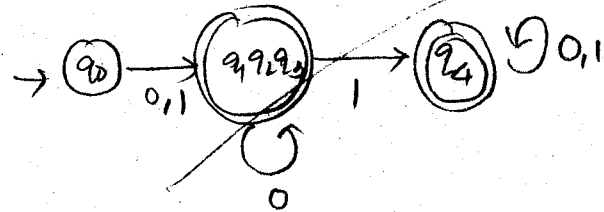


$D: \{q_1, q_4\}$
 $\{q_2, q_4\}$
 $\{q_3, q_4\}$
 $\{q_0, q_4\}$

$ID: \{q_1, q_3\}$
 $\downarrow \quad \downarrow$
 $\{q_2\} \quad \{q_4\}$

	q_0	q_1	q_2	q_3	q_4
q_0		D	D	D	D
q_1			ID	ID	D
q_2				ID	D
q_3					D
q_4					

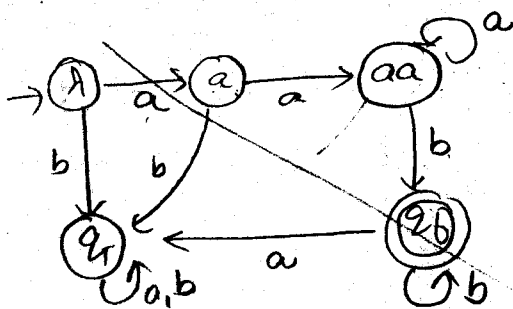
$\{q_0\}, \{q_1, q_2, q_3\}, \{q_4\}$



EXERCISES

② (a) find minimal dfa for

$$L = \{a^n b^m : n \geq 2, m \geq 1\}$$

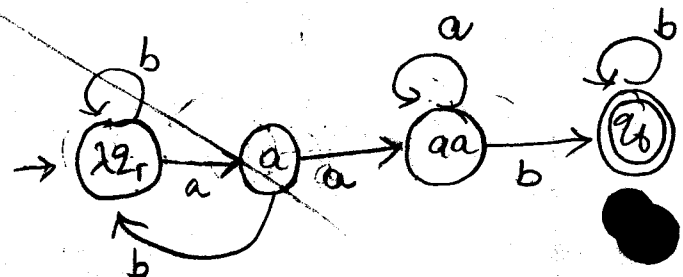


$ID: (aa, q_f)$
 (λ, q_1)

$D: (a, q_f) \quad (\lambda, q_f) \quad (q_1, aa)$
 $(q_1, q_f) \quad (\lambda, aa) \quad (q_f, aa)$

	λ	a	aa	q_1	q_f
λ		D	D	D	ID
a			D	D	D
aa				ID	D
q_1					D
q_f					

$\{\lambda, q_1\}, \{a\}, \{aa\}, \{q_f\}$

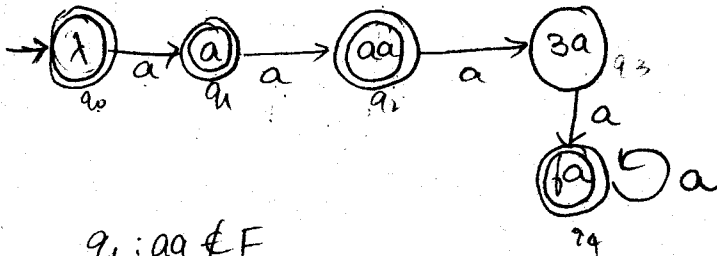


(a) $L = \{a^n b^m : n \geq 2, m \geq 1\}$

minimal

(b) $L = \{a^n b : n \geq 0\} \cup \{b^n a : n \geq 1\}$

(c) $L = \{a^n : n \geq 0, n \neq 3\}$



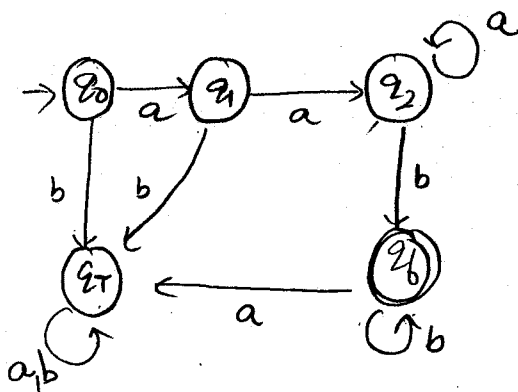
$q_1 : aa \notin F$
 $(q_4, aa) \in F : (q_1, q_4) D$

$(\lambda, aaa) \notin F : (\lambda, 3a),$
 $(3a, aaa) \in F : D$

	λ	a	aa	3a	4a
λ	D	D	D	D	
a		D	D	D	
aa			D	D	
3a				D	
4a					D

already in minimal //

(a)



$q_4 \in F$
 $rest \notin F : D\text{-states}$

	q_0	q_1	q_2	q_3	q_4
q_0	D	D	D	D	
q_1		D	D	D	
q_2			D	D	
q_3				D	
q_4					D

\therefore minimal

CHAPTER : 3

RL \neq RQ

$$\rightarrow L(r_1 + r_2) = L(r_1) \cup L(r_2)$$

$$L(r_1 r_2) = L(r_1) \cdot L(r_2)$$

$$L(r_1^*) = (L(r_1))^*$$

Example
3.2

$$L(a^* \cdot (a+b)) ?$$

$$\begin{aligned} L(a^* \cdot (a+b)) &= L(a^*) \cdot L(a+b) \\ &= L(a^*) \cdot \{L(a) \cup L(b)\} \\ &= L(a^*) \cdot \{a, b\} \\ &= \{\lambda, a, aa, \dots\} \cdot \{a, b\} \\ &= \{a\}^+ \cup \{a^n b^{m+1} : n \geq 0\} \end{aligned}$$

Example
3.3

$$\Sigma = \{a, b\}$$

$$r = (a+b)^* \cdot (a+bb)$$

$$\{a, b\}^* \cdot \{a, bb\} = \{a, bb, aa, abb, ba, bbb, \dots\}$$

Example
3.4

$$r = (aa)^* (bb)^* b$$

$$L = \{(aa)^n (bb)^m b : n, m \geq 0\}$$

$$L = \{a^{2n} b^{2m+1} : n, m \geq 0\}$$

Example
3.5

$$\Sigma = \{0, 1\} : \text{w has at least one pair of consecutive zeroes.}$$

$$(0+1)^* 00 (0+1)^*$$

$$(1+01)^* (0+1)$$

3.6. no consecutive zeroes:

①

$L \{ (a+b)^* b (a+ab)^* \}$ Find strings $|w| < 4$.

$\{ \lambda, a, b, ab, ba, aa, bb, aba, baa, aaa, bba, bab, abb, aab, \dots \} \cdot b$.

$\{ \lambda, a, ab, aab, aba, aaa \} - \dots$

$|w| < 4$:

$\{ b, ab, bb, ba, bab, \dots \}$

②

$((0+1)(0+1)^*)^* 00 (0+1)^*$ denote at least one pair of consecutive 0's.
yes.

③

$r = (1+01)^* (0+1)^*$ also denotes no consecutive zeroes.

↓

$(1+01)^* (0 + \lambda + \{1\}^+)$

$((1+01)^* (0 + \lambda)) + (1+01)^* \{1\}^+$

↓

$(1+01)^*$



$= (1+01)^* (0 + \lambda) \Rightarrow$ no consecutive zeroes

④

RE? $\{a^n b^m : n \geq 3, m \text{ is even}\}$

$aaa(a^*)(bb)^*$

⑤

RE=? $\{a^n b^m : (n+m) \text{ is even}\}$

$\left[(aa)^*(bb)^* + (aa)^*a(bb)^*b \right]$

RE = ?

⑥

(a)

$$L_1 = \{a^n b^m : n \geq 4, m \leq 3\}$$

$$aaaa \cdot a^* (\lambda + b + bb + bbb)$$

(b) $L_2 = \{a^n b^m : n < 4, m \leq 3\}$

$$(\lambda + a + aa + aaa) (\lambda + b + bb + bbb)$$

(c) $\bar{L}_1 : \{a^n b^m : n < 4, m > 3\}$ (d)

$$\times (\lambda + a + aa + aaa) bbbb b^* +$$

either $n < 4$ or $m \geq 4$ (or) $a^k b^l$

$$(\lambda + a + aa + aaa) b^* + a^* bbbbbb^* +$$

$$(a+b)^* b a (a+b)^*$$

(d) $\bar{L}_2 : \{a^n b^m : n < 4, m \leq 3\}$

$$n \geq 4 / m > 3$$

$$\bar{L}_2 : aaaaa^* b^* + a^* bbbb b^* + (a+b)^* b a (a+b)^*$$

⑦

$$L [(aa)^* b (aa)^* + a(aa)^* b a(aa)^*]$$

$$wbw : w : a^{2n} : n \geq 0$$

$$w : a^{2n+1} : n \geq 0$$

b having even a's on both ends or
b having odd a's on both ends.

RE(L) =
all possible
rule
breakers
in
RE(L)
→

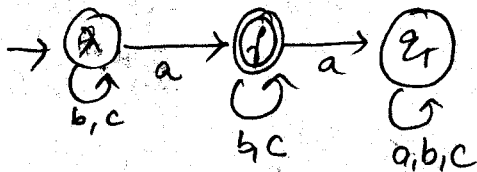
(16)

(a) $\Sigma = \{a, b, c\}$

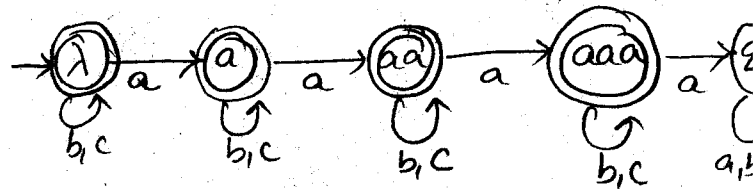
Exactly one a.

$$(b+c)^* a (b+c)^*$$

a ✓ bcbca ✓
ab ✓ bcaaa ✗



(b) no more than 3 a's.



$$\begin{aligned} &[(b+c)^* a (b+c)^* a (b+c)^* a (b+c)^* \\ &+ (b+c)^* + (b+c)^* a (b+c)^* + \\ &(b+c)^* a (b+c)^* a (b+c)^*] \end{aligned}$$

Test: aa ✓ aaaa ✗
bca ✓

(c) @least one occurrence of each symbol in Σ

$$(a+b+c)^* a (a+b+c)^* b (a+b+c)^* c (a+b+c)^*$$

(d) no run of a's $|w|_a > 2$

0, 1, 2

$$(\lambda + a + aa + b + c)^*$$

$$(b+c)^* (\lambda + a) (b+c)^* (\lambda + a) (b+c)^*$$

(e) run's of a's are multiples of 3.

$$(b+c)^* aaaa^* (b+c)^* aaaa^* (b+c)^*$$

$$(\lambda + aaaa^* + b + c)^*$$

(17) @ Ending in 01

$$(0+1)^* 01$$

@ Even no. of zeroes

$$[1^* 01^* 01^* + 1]^*$$

(17) (b) Not Ending in 01:

$$1^* (0+01+11)^* 11^* (0+\lambda)$$

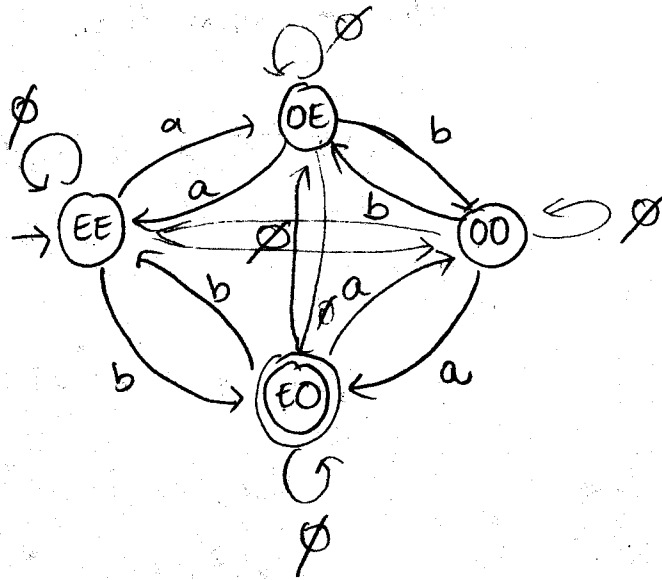
RE = ?

$L = \{ w \in \{a,b\}^+ : n_a(w) \text{ is even, } n_b(w) \text{ is odd} \}$

sample 3.11

FIND RE for a DFA?

to all unknown moves!



$$r + \emptyset = r$$

$$r\emptyset = \emptyset$$

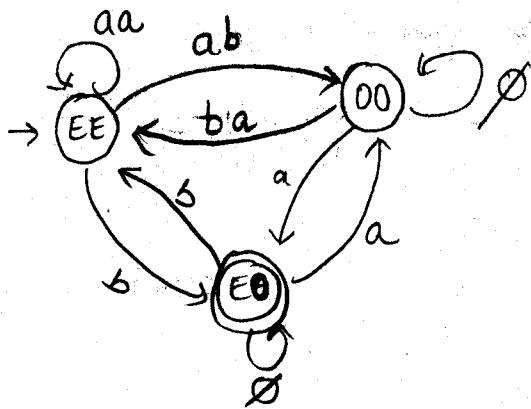
$$\emptyset^* = \lambda$$

step 1

$$r_{EE} = \emptyset + a\emptyset^*a = aa$$

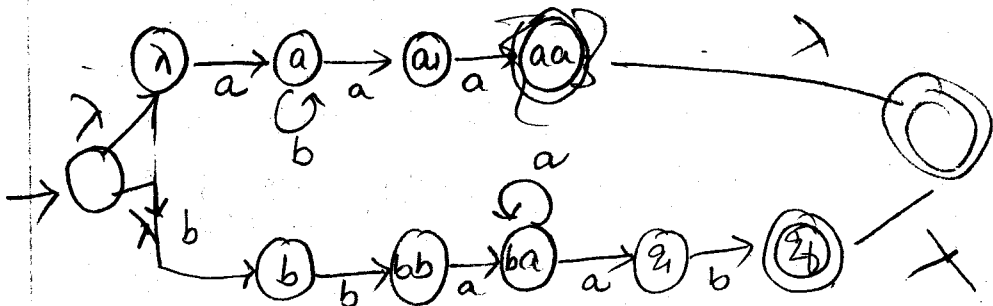
step 2

now 3-state Rule

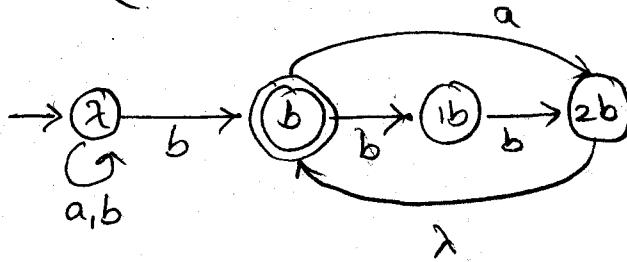


EXERCISE

$L(ab^+aa + bba^+ab)$:

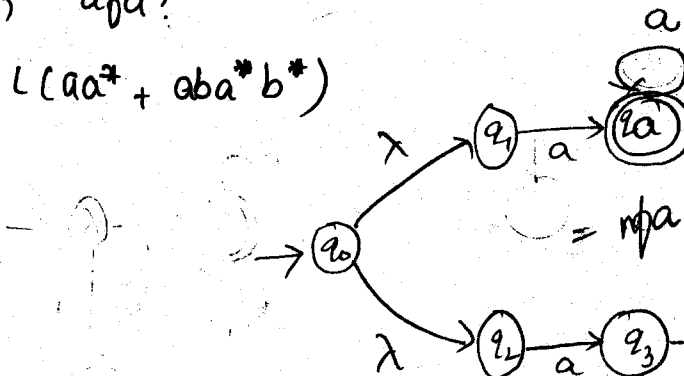


③

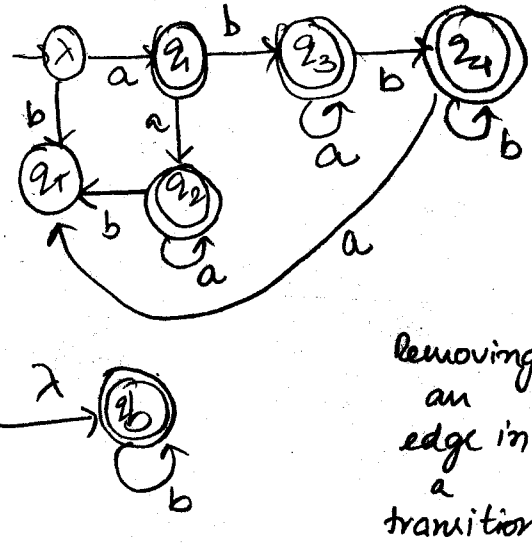
nfa? $((a+b)^* b (a+bb)^*)$ 

④

a) dfa?

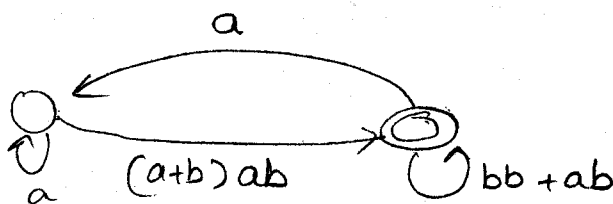
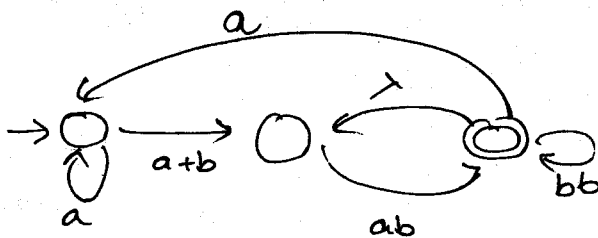
 $L(aa^* + aba^*b^*)$ 

dfa:



Removing
an
edge in
a
transition
graph
($q_0 \notin F$)

⑤

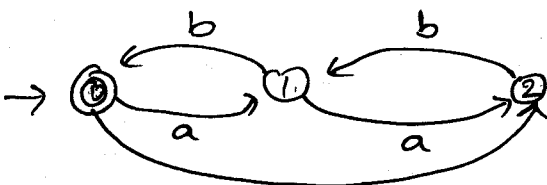
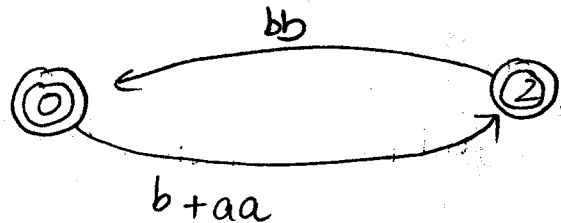
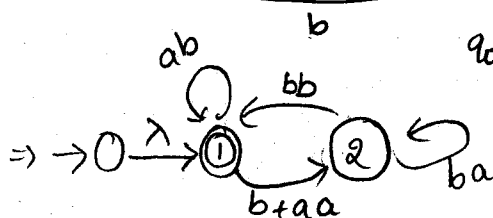


$$r = a^* (a+b) ab (ab + bb + aa^* (a+b) ab)^*$$

aabb bbb b

⑩

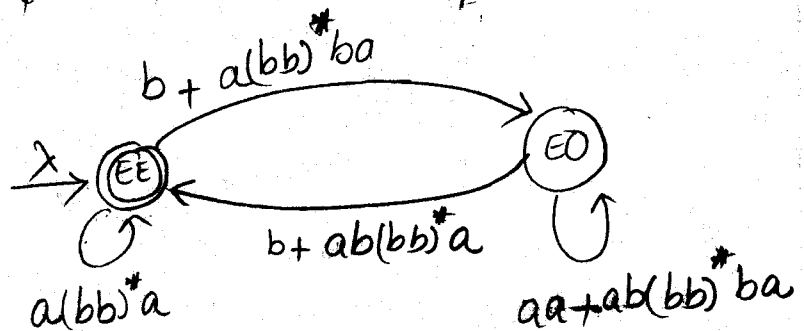
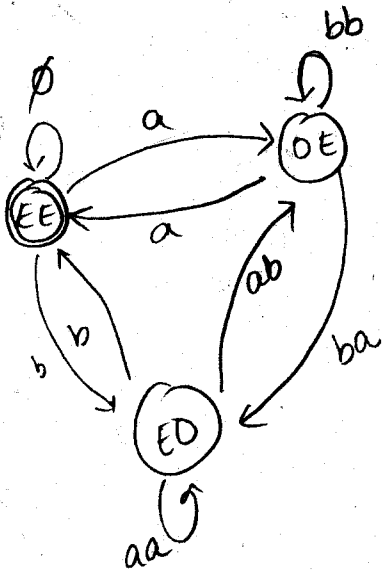
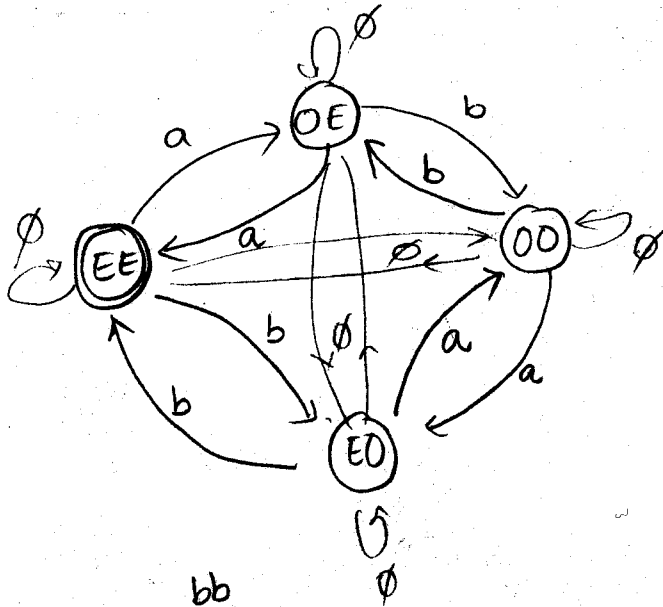
(b)

 $q_0 \notin F$  $b + aa$ 

$$ab + (b+aa)(ba)^* bb$$

13

EE=? on $\Sigma = \{a, b\}$
 (a) $L = \{w : n_a(w), n_b(w) \text{ are even}\}$



RE:

$$\lambda + [a(bb)^*a][b + a(bb)^*ba][aa + ab(bb)^*ba][b + ab(bb)^*a]^*$$

$\lambda \checkmark$
 aa

CH # 3.3

1

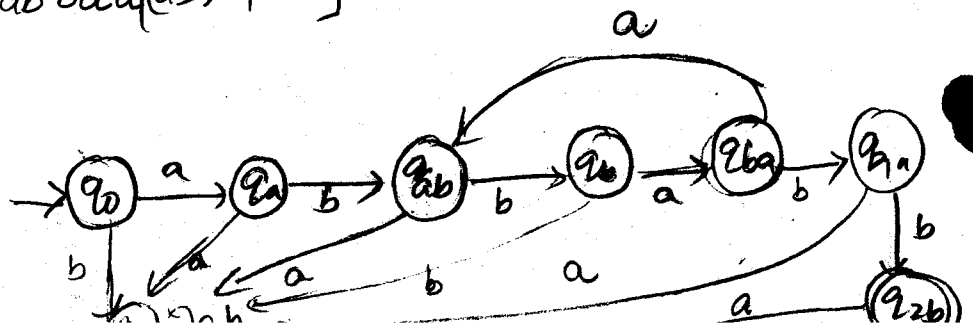
dfa=?

$S \rightarrow abA$

$A \rightarrow baB$

$B \rightarrow aA/bb$

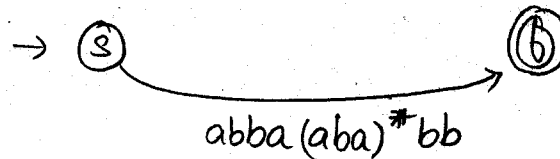
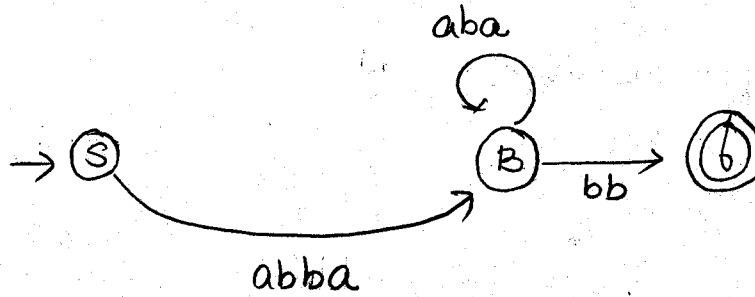
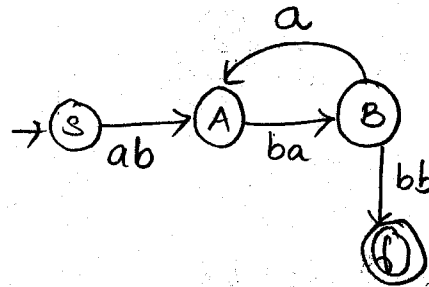
$$abbaa[(ab)^* + bb]$$



3

 $u_a = ?$ for 1 $S \rightarrow abA$ $A \rightarrow baB$ $B \rightarrow aA/bb$

RE =



Test: abbabbb ✓

abbaabaababbv ✓

 $S \rightarrow ABbb$ $A \rightarrow abba$ $B \rightarrow Baba / \lambda$ $S \rightarrow ABbb \rightarrow abbaBbb \rightarrow$ $abbaBaba bb \rightarrow$ $abba(aba)^*bb \checkmark$

4

 $RLQ, LLQ = ?$ $\{a^n b^m : n \geq 2, m \geq 3\}$

RLQ:

 $S \rightarrow aaAB$ $A \rightarrow aA/\lambda$ $B \rightarrow bbbC$ $C \rightarrow bC/\lambda$

LQ

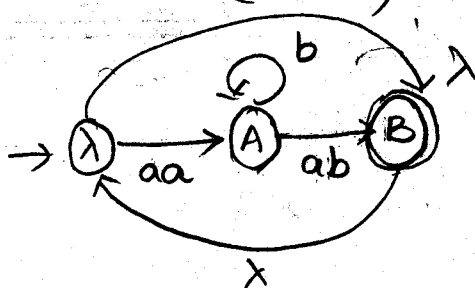
 $S \rightarrow aaA bbbB$ $A \rightarrow aA/\lambda$ $B \rightarrow bB/\lambda$

LLQ:

 $S \rightarrow ABbbb$ $A \rightarrow Caa$ $C \rightarrow Ca/\lambda$ $B \rightarrow Bb/\lambda$

RLG = ?

$(aab^*ab)^*$



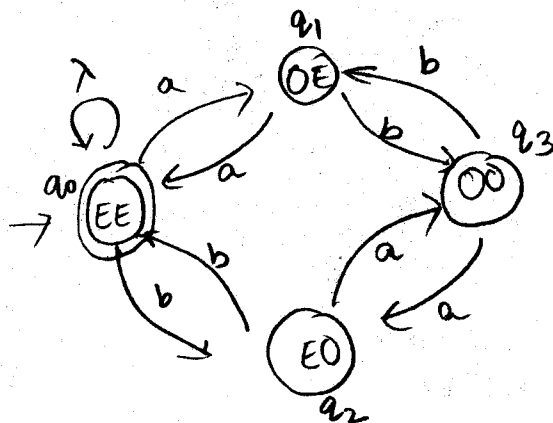
$S \rightarrow aaA / \lambda$

$A \rightarrow bA / abS$

13

(a) ~~REG~~ RG = ? $\Sigma = \{a, b\}$

(a) $n_a(w), n_b(w)$ are even.



$q_0 \rightarrow aq_1 / \lambda / bq_2$

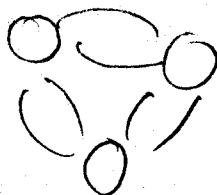
$q_1 \rightarrow bq_3 / aq_0$

$q_2 \rightarrow aq_3 / bq_0$

$q_3 \rightarrow aq_2 / bq_1$

$$(n_a - n_b) \bmod 3 = 1$$

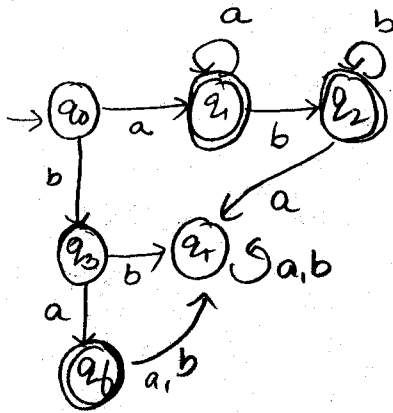
draw



Right Quotient: L_1/L_2

$L_1: \{a^m b^n : n \geq 1, m \geq 0\} \cup \{ba\}$

$L_2: \{b^m : m \geq 1\}$



for L_1/L_2

Final states are:

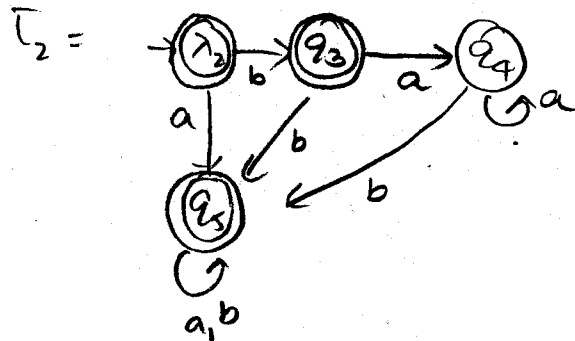
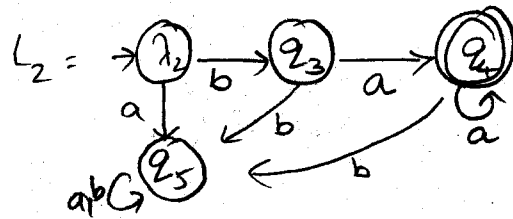
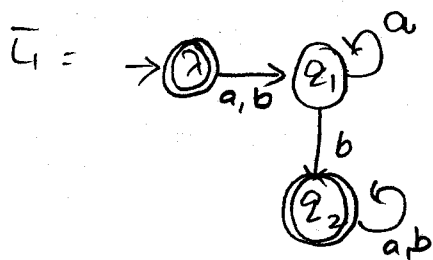
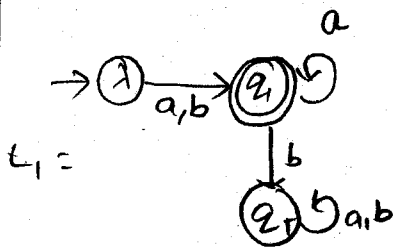
q_1, q_2

RIGHT
 QUOTIENT
 L_1/L_2
 draw dfa
 for L_1
 → 4 nodes
 apply L_2
 to check final
 state

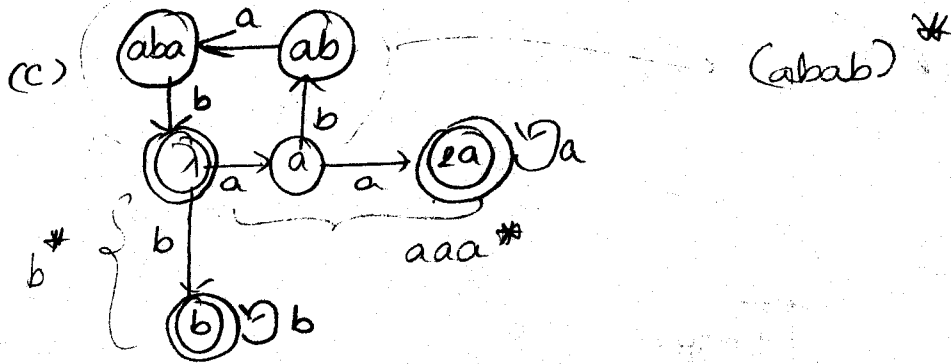
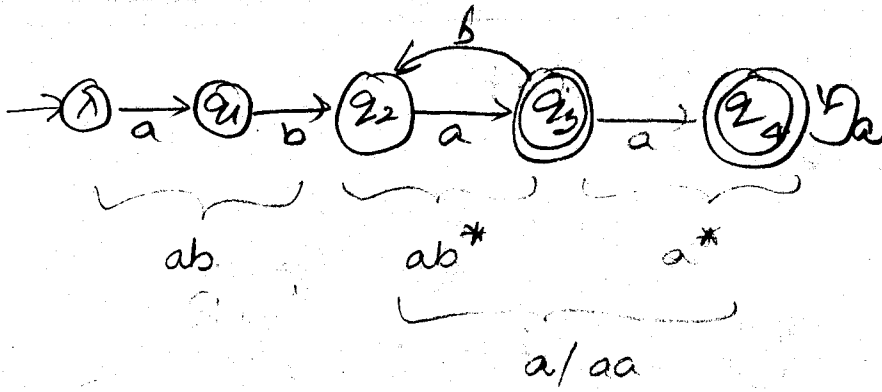
Exercises

(a) $(a+b)a^* \cap (baa^*)$

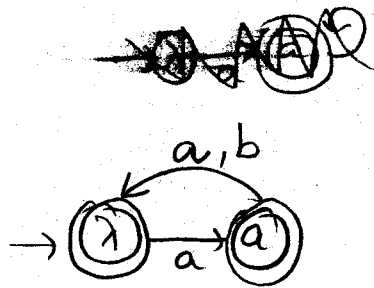
$nb a = ?$



④ (b) $ab(ab)^*(a+aa)$



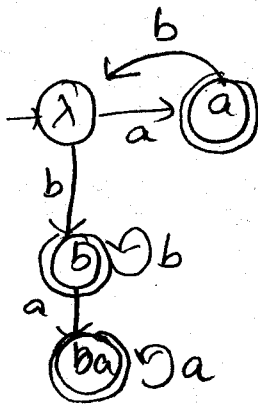
(d)



$\lambda, a, aa, ab, abab$
 (aa^*b)

⑥

⑤ (a)



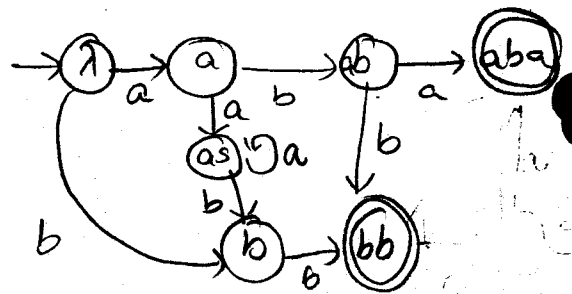
ab^*a^*
 $a \checkmark$
 $aba \checkmark$
 $abba \checkmark$
 ab^*a
 abb

⑦

$bb \checkmark$
 $abb \checkmark$
 a^*bb

ab^*ba
 $aba \checkmark$
 $abba \checkmark$
 $abba \checkmark$

a^*bba



A Grammar $G = (V, T, S, P)$ is CF if all productions in P are of the form

$$A \rightarrow x$$

$$(x \in (V \cup T)^*, A \in V)$$

Example
5.1

$$G = (\{S\}, \{a, b\}, S, P)$$

$$P: \begin{aligned} S &\rightarrow aSa \\ S &\rightarrow bSb \\ S &\rightarrow \lambda \end{aligned}$$

$$S \rightarrow aSa \rightarrow abba$$

$$S \rightarrow aSa \rightarrow aaSaa \rightarrow aabbbaa$$

$$L(G) = \{ww^R : w \in \Sigma^*\}$$

Example
5.2

$$G: P:$$

$$\begin{aligned} S &\rightarrow abB \\ A &\rightarrow aaBb \\ B &\rightarrow bbAa \\ A &\rightarrow \lambda \end{aligned}$$

$$S \rightarrow abbbAa \rightarrow \boxed{abbbba}(ba)^0$$

$$\rightarrow abbbbaaBba \rightarrow \boxed{abbbbaaabbba}$$

$$\rightarrow abbbbaaabbbaaBbaba \rightarrow \boxed{abbbbaaabbbaaabbbaa}$$

$$\rightarrow abbbbaaabbbaaabbbaaAababa \rightarrow \boxed{abbbbaaabbbaaabbbaaabbbaa}$$

$$L(G) = \{ab(bbba)^n bba(ba)^m : n, m \geq 0\}$$

Example
5.3

ST $L = \{a^n b^m : n \neq m\}$ is context free.

$$\left(\begin{aligned} L: & \{a^n b^m : n \geq 0\} \\ S & \rightarrow asb / \lambda \end{aligned} \right)$$

$$S \rightarrow aSb / aA / bB$$

$$A \rightarrow aA / \lambda$$

$$B \rightarrow bB / \lambda$$

$$S \rightarrow AS_1 / BS_1$$

$$A \rightarrow aA / \lambda$$

$$B \rightarrow bB / \lambda$$

$$S_1 \rightarrow aS_1b / \lambda$$

Example
5.4

$$S \rightarrow asb / SS / \lambda$$

$$L(G) = \{ w : w \in \{a,b\}^*, n_a(w) = n_b(w) \text{ \& } n_a(\gamma) \geq n_b(\gamma), \gamma \text{ is any prefix of } w \}$$

$$n_a(\gamma) \geq n_b(\gamma), \gamma \text{ is any prefix of } w \}$$

$$S \rightarrow asb \rightarrow aaSbb \rightarrow aabb$$

$$S \rightarrow SS \rightarrow asbaSb \rightarrow abab \dots$$

Leftmost & Right Most derivations:

$$S \rightarrow AB$$

$$A \rightarrow aaA$$

$$A \rightarrow \lambda$$

$$B \rightarrow Bb / \lambda$$

$$S \rightarrow aaABb$$

$$A \rightarrow aaA / \lambda$$

$$B \rightarrow Bb / \lambda$$

$$L(G) = \{ a^{2n}b^m : n, m \geq 0 \}$$

Example
5.5

$$S \rightarrow aAB$$

$$A \rightarrow bBb$$

$$B \rightarrow A / \lambda$$

$$S \rightarrow aAB \rightarrow abBb \rightarrow \text{abb}$$

$$S \rightarrow aAB \rightarrow abBb bBb \rightarrow \text{abbbbb}$$

$$S \rightarrow aAB \rightarrow abBb bBb \rightarrow \text{abbbbbBbb} \rightarrow \text{abbbbbbb}$$

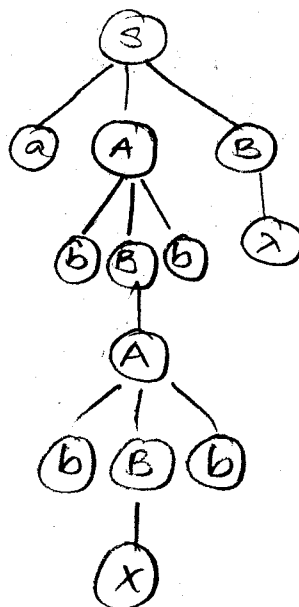
$$L(G) = \{ ab^{2^n} : n \geq 0 \}$$

Example
5.6

$$S \rightarrow aAB$$

$$A \rightarrow bBb$$

$$B \rightarrow A / \lambda$$



CHAPTER: 5-1
(CONTEXT FREE GRAMMARS)

Prathima Bhima NOV 3

Def: $G = (V, T, S, P)$
 $A \rightarrow \alpha$
 $\alpha \in (VUT)^*$

Eg: 5.1

$G = (\{S\}, \{a, b\}, S, P)$

$S \rightarrow aSa$

$S \rightarrow bSb$

$S \rightarrow \lambda$

$L(G) = \{ww^R : w \in \{a, b\}^*\}$

G is CFG, but not regular.

Eg: 5.2

$S \rightarrow abB$

$A \rightarrow aaBb$

$B \rightarrow bbAa$

$A \rightarrow \lambda$

$S \Rightarrow abbbAa \Rightarrow \underline{abbb}a$

$\Rightarrow \underline{abbb}a\underline{bb}ba$

$\Rightarrow abbbbaabbAaba \Rightarrow \underline{abbb}ba\underline{abb}ba\underline{ab}ba$

$L(G) = \{ab(bbaa)^n bba(ba)^n : n \geq 0\}$

Eg: 5.3

$L = \{a^n b^m : n \neq m\}$

$\left(\begin{array}{c} n=m \\ S \rightarrow aSb / \lambda \end{array} \right)$

$n > m$

$a^{n+x} b^m$

$S_1 \rightarrow AB$

$A \rightarrow aA / a$

$B \rightarrow aBb / \lambda$

$n < m, m \geq n$

$a^n b^{n+x}$

$S_2 \rightarrow BC$

$C \rightarrow Cb / b$

$S \rightarrow AB / BC$

$B \rightarrow aBb / \lambda$

$A \rightarrow aA / a$

$C \rightarrow bC / b$

$G = (V, T, S, P)$

Ex. 5.4

$$S \rightarrow aSb / SS / \lambda$$

$$\Rightarrow L(a) = \{ w \in \Sigma_{a,b}^* : n_a(w) = n_b(w), n_a(v) \geq n_b(v) \text{ where } v \text{ is prefix of } w \}$$

$$S \rightarrow AB$$

$$A \rightarrow aAa / \lambda$$

$$B \rightarrow Bb / \lambda$$

$$L = \{ a^{2n} b^m : m, n \geq 0 \}$$

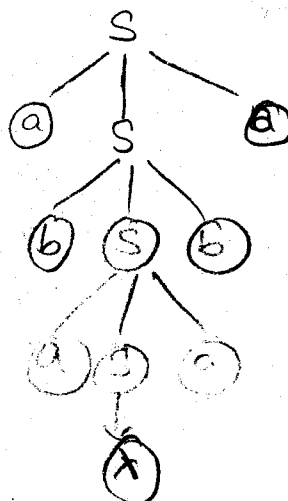
(EXERCISES)

②

$$S \rightarrow aSa$$

$$S \rightarrow bSb$$

$$S \rightarrow \lambda$$



⑦

Find CFG for $n \geq 0, m \geq 0$

⑧

$$L = \{ a^n b^m : n \leq m+3 \}$$

$$n=m$$

$$\left\{ \begin{array}{l} S \rightarrow aSb / \lambda \end{array} \right\}$$

$$① \quad n = m+3 \rightarrow$$

$$② \quad n < m+3 \Rightarrow \text{add any no. of } b's$$

↓

$$S \rightarrow aaaaA$$

$$A \rightarrow aAb / B$$

$$B \rightarrow bB / \lambda$$

$$n \leq m+3 \Rightarrow n=0,1,2$$

$$S \rightarrow aA / aaA / aaaaA / \lambda$$

$$A \rightarrow aAb / B$$

$$B \rightarrow bB / \lambda$$

$$n=m+3$$

$$a^n b^m \Rightarrow a^{m+3} b^m$$

$$S \rightarrow AB$$

$$A \rightarrow aaaa$$

$$B \rightarrow aBb / \lambda$$

→

$$\left\{ \begin{array}{l} S \rightarrow aaaaA \\ A \rightarrow aAb / \lambda \end{array} \right\}$$

$$S \rightarrow aSb / aaaa$$

$$m=0 : n=0 \quad n=1 \quad n=2 \quad n=3$$

$$\lambda \checkmark \quad a \checkmark \quad aa \checkmark \quad aaa \checkmark$$

$$m=1 : n=1 \quad n=2 \quad n=3 \quad n=4$$

$$ab \checkmark \quad aab \checkmark \quad aaab \checkmark \quad aaabb \checkmark$$

$$m=1 \quad n=5 \quad X$$

$$aaaaab$$

$$aaaaA \rightarrow aaaa$$

CH: 5.1
EXERCISES.

7 (b)

HW

$$L = \{a^n b^m : n \neq m-1\}$$

$$n = m-1$$

$$m = n+1$$

$$S \rightarrow Ab$$

$$A \rightarrow aAb/\lambda$$

$$n=0 \ m=1 : b \checkmark$$

$$n=1 \ m=2 : abb \checkmark$$

$$n=1 \ m=1 : ab \times$$

$$n < m-1$$

add b's

$$S \rightarrow Ab$$

$$A \rightarrow aAb/B$$

$$B \rightarrow bB/b$$

$$n > m-1$$

add a's

$$S \rightarrow Ab$$

$$A \rightarrow aAb/C$$

$$C \rightarrow aC/a$$

Test: $n: 0, 1, 2 : m=4$

$$\checkmark \begin{cases} bbbb : S \rightarrow Ab \rightarrow Bb \rightarrow bbbb \checkmark \\ abbbb : S \rightarrow Ab \rightarrow aAb \rightarrow aBb \rightarrow abbbb \checkmark \\ aabbbb : S \rightarrow Ab \rightarrow aaAb \rightarrow aabbbb \checkmark \\ aaabbbb : S \rightarrow Ab \rightarrow aaaAb \rightarrow aaabbbb \checkmark \\ aaaaabbbb : S \rightarrow Ab \rightarrow aaaaAb \rightarrow aaaaabbbb \times \\ aaaaaabbbb : S \rightarrow aaaaaAb \rightarrow aaaaaabbbb \times \end{cases}$$

Test $m=3 \ n: 3, 4, 5, 6 \dots \checkmark$

$$aaabbbb : S \rightarrow aaAb \rightarrow aaabbb \checkmark$$

$$aaaaabbbb : aaAb \rightarrow aaaaCbb \rightarrow aaaaabbbb \checkmark$$

$$\% \quad n \neq m-1$$

\Rightarrow

$$S \rightarrow Ab$$

$$A \rightarrow aAb/B/C$$

$$B \rightarrow bB/b$$

$$C \rightarrow aC/a$$

$$CFG = (\quad)$$

Test

HW

7. (c)

$$L = \{a^n b^m : n \neq 2m\}$$

$$\left(\begin{array}{l} n = 2m \\ S \rightarrow aaSb / \lambda \end{array} \right)$$

aaabbbn: even

$$S \rightarrow aaSb / \lambda$$

add a's / b's

$$S \rightarrow aaSb / A / B$$

$$A \rightarrow aA / a$$

$$B \rightarrow bB / b$$

n: odd

$$S \rightarrow aaS / aB$$

$$B \rightarrow bB / \lambda$$

$$S \rightarrow S_1 / S_2$$

$$S_1 \rightarrow aaS_1 b / A / B$$

$$A \rightarrow aA / a$$

$$B \rightarrow bB / b$$

$$S_2 \rightarrow aaS_2 / aC$$

$$C \rightarrow bC / \lambda$$

$$S \rightarrow \epsilon / 0$$

$$E \rightarrow aaEb / \lambda$$

and with more a's or more b's.

$$0 \rightarrow aa0 / aE$$

$$C \rightarrow bC / \lambda$$

 \Rightarrow

$$E \rightarrow aaEb / A / B$$

$$A \rightarrow aA / a$$

$$B \rightarrow bB / b$$

$$\{ \lambda \notin L(G) \}$$

$$\therefore \text{CFG} = (\dots)$$

Test: - - -

CH # 5.1
(EXERCISES)

Prathima Bhima

NOV 5

7) HW d.

$$L = \{a^n b^m : 2n \leq m \leq 3n\}$$

$$\Rightarrow m = 2n \mid m = 3n$$

$$\therefore S \rightarrow aSbb \mid aSbbb \mid \lambda$$

(e) HW $L = \{w \in \{a,b\}^* : n_a(w) \neq n_b(w)\}$

$$\left(\begin{array}{l} n_a(w) = n_b(w) \\ S \rightarrow SS \mid aSb \mid bSa \mid \lambda \end{array} \right)$$

add a's or add b's \Rightarrow

$$S \rightarrow SS \mid aSb \mid bSa \mid a \mid bS \mid a/b$$

(f) HW $L = \{w : w \in \{a,b\}^* : n_a(v) \geq n_b(v) : v \text{ is prefix of } w\}$

$$S \rightarrow SS \mid aSb \mid \lambda$$

(g) HW $L = \{w \in \{a,b\}^* : n_a(w) = 2n_b(w) + 1\}$

$$n_a(w) = 2n_b(w)$$

$$S \rightarrow SS \mid aaSb \mid bSaa \mid aSba \mid aSab \mid abSa \mid baSa \mid \lambda$$

$$n_a(w) = n_b(w) + 1$$

$$S \rightarrow SS \mid aaSb \mid aSba \mid aSab \mid bSaa \mid baSa \mid abSa \mid a$$

Test:

aaab : $S \rightarrow aaSb \rightarrow aaab$ ✓

aab : $S \rightarrow aSab \rightarrow \times$

$$n \geq 0, m \geq 0, k \geq 0.$$

$$(a) L = \{ a^n b^m c^k : n=m \text{ or } m \leq k \}$$

$$n=m, \textcircled{K}$$

$$\begin{aligned} S_1 &\rightarrow AB \\ A &\rightarrow aAb/\lambda \\ B &\rightarrow cB/\lambda \end{aligned}$$

$$m \leq k, \textcircled{M}$$

$$\begin{aligned} S &\rightarrow AB \\ A &\rightarrow aA/\lambda \\ B &\rightarrow bBC \\ C &\rightarrow cC/\lambda \end{aligned}$$

$$\begin{aligned} S_2 &\rightarrow DE \\ D &\rightarrow aD/\lambda \\ E &\rightarrow bEF \\ F &\rightarrow cF/\lambda \end{aligned}$$

$$S \rightarrow S_1 / S_2$$

$$n=m \textcircled{K}$$

$$\begin{aligned} S_1 &\rightarrow AB \\ A &\rightarrow aAb/\lambda \\ B &\rightarrow cB/\lambda \end{aligned}$$

$$m \leq k \textcircled{M}$$

$$\begin{aligned} S_2 &\rightarrow CD \\ C &\rightarrow aC/\lambda \\ D &\rightarrow bDC/E \\ E &\rightarrow cE/\lambda \end{aligned}$$

$$\begin{array}{l} m \leq k \\ m = k \\ \hline X \rightarrow bXc \\ \text{add c's} \\ X \rightarrow bXc/c \\ C \rightarrow cC/\lambda \end{array}$$

HW
(b)

$$L = \{ a^n b^m c^k : n=m \text{ or } m \neq k \}$$

$$S \rightarrow S_1 / S_2$$

$$\begin{aligned} S_1 &\rightarrow AB \\ A &\rightarrow aAb/\lambda \\ B &\rightarrow cB/\lambda \end{aligned}$$

$$\begin{array}{l} m = k \\ \hline X \rightarrow bXc/\lambda \\ \text{add b's / c's} \\ X \rightarrow bXc/Y/Z \\ Y \rightarrow bY/b \\ Z \rightarrow cZ/c \end{array}$$

$$\begin{aligned} m &\neq k \\ S_2 &\rightarrow CD \\ C &\rightarrow aC/\lambda \\ D &\rightarrow bDC/E/F \\ E &\rightarrow bE/b \\ F &\rightarrow cF/c \end{aligned}$$

HW
(c)

$$L = \{ a^n b^m c^k : k = n+m \}$$

$$\underbrace{aa}_{n} \underbrace{abb}_{m} \underbrace{b-bcc}_{n+m} \underbrace{c}_{1}$$

for every 'a' add a 'c'
for every 'b' add a 'c'

$$\begin{aligned} S &\rightarrow aSc / B \\ B &\rightarrow bSc / \lambda \end{aligned}$$

$$G = (\{S, B\}, \{a, b, c\}, S, P)$$

(d)
HW

$$L = \{a^n b^m c^k : n + 2m = k\}$$

aaa... aabb... bbcc... c
 n m n+2m

Every a add one c
 Every b add 2 c's

$$S \rightarrow aSc / B$$

$$B \rightarrow bBcc / \lambda$$

n:0 λ ✓
 m:0
 k:0 ac
 abccc ✓

(e)
HW

$$L = \{a^n b^m c^k : k = |n - m|\}$$

a... ab... bc... c
 n m excess a's or b's in the string so far.

$$k = n - m$$

$$n = m + k$$

$$k = m - n$$

$$m = n + k$$

$$S_1 \rightarrow aS_1c /$$

$$\rightarrow a b / \lambda$$

$$S_2 \rightarrow aS_2b / B$$

$$B \rightarrow bBc / \lambda$$

$$S \rightarrow S_1 / S_2$$

HW

$$(f) L = \{w \in \Sigma^* : n_a(w) + n_b(w) \neq n_c(w)\}$$

$$n + m < k$$

\Rightarrow add any no. of c's

$$n + m > k$$

add any no. of a's or b's or both

$$S \rightarrow S_1 / S_2$$

HW
 (8) (9) $L = \{a^n b^m c^k, k \neq n+m\}$

$$\left(\begin{array}{l} k = n+m \\ S \rightarrow aSc/B \\ B \rightarrow bBc/\lambda \end{array} \right)$$

$$(S \rightarrow S_1/S_2)$$

$n+m < k$
 add any no. of c's

$$\left(\begin{array}{l} n=m=k \\ S \rightarrow aSc/B \\ B \rightarrow bBc/\lambda \end{array} \right)$$

$$\boxed{\begin{array}{l} S \rightarrow aSc/cS/c/B \\ B \rightarrow bSc/\lambda \end{array}}$$

Test abcc: $aSc \rightarrow abSc \times$

abccc ✓

acc ✓

Test

a: $S \rightarrow a \checkmark$

aabccc: $aSc \rightarrow aaSc \rightarrow aabccc \times$

aabcc: $aSc \rightarrow aaSc \rightarrow aabcc \checkmark$

$n+m > k$

~~$\{a, b, aa, ab, ba, bb, abc, \dots\}$~~

$$\left(\begin{array}{l} n=m=k \\ S \rightarrow aSc/B \\ B \rightarrow bBc/\lambda \end{array} \right)$$

add atleast one a or more
 add atleast one b or more

$$\Rightarrow \boxed{\begin{array}{l} S_2 \rightarrow aS_2c/aS_2/bS_2/a/b/B \\ B \rightarrow bBc/\lambda \end{array}}$$

⑧

(h) HW $L = \{a^n b^n c^k : k \geq 3\}$

$$\left(\begin{array}{l} a^n b^n c^k : n, k \geq 0 \\ S \rightarrow AB \\ A \rightarrow aAb / \lambda \\ B \rightarrow cB / \lambda \end{array} \right)$$

 $k \geq 3$ $B \rightarrow cB / ccc$

↓

minimum 3 c's or more

$$\begin{aligned} \therefore S &\rightarrow AB \\ A &\rightarrow aAb / \lambda \\ B &\rightarrow cB / ccc \end{aligned}$$

Testccc : $S \rightarrow AB \rightarrow B \rightarrow ccc$ ✓abccc : $S \rightarrow abB \rightarrow abccc$ ✓9. ST $L = \{w \in a, b, c^* : |w| = 3n, a(w)\}$ is a CFG.
$$\begin{array}{l} a^n \\ b^m \\ c^k \end{array}$$

$$n+m+k = 3n.$$

$$m+k = 2n$$

∃ for every b an a
 ∃ for every c an a

(no order $\therefore \Sigma^*$)

$$\begin{aligned} \therefore S &\rightarrow SS / aSc / B \\ B &\rightarrow bBc / cBb / \lambda \end{aligned}$$

Test

$$\begin{aligned} S &\rightarrow aSX / bSY / cSZ \\ X &\rightarrow bc / cb / bb / cc \\ Y &\rightarrow ac / ca / ab / ba \\ Z &\rightarrow abba / ac / ca \end{aligned}$$

sub are

$$m+k = 2n$$

$\underbrace{aaa}_n \underbrace{bb \dots b}_m \underbrace{ccc}_{2n}$

$S \rightarrow ABC$

$A \rightarrow$

$$\underline{m+k = 2n}$$

$$2 [n_a(w)] = n_b(w) + n_c(w)$$

Every a has 2 more symbols
 \downarrow
 Either [b/c]

$\Rightarrow S \rightarrow aSX / bSY / cSZ / SS / \lambda$
 $X \rightarrow bb / cc / bc / cb \rightarrow \text{already } / a$
 $Y \rightarrow ab / ba / ac / ca$
 $Z \rightarrow ac / ca / ab / ba$

Test:
 $n_a(w) = 1$
 $3n_a(w) = 3$

$w = abc$
 $w = bac$

$S \rightarrow aSX \rightarrow abc \checkmark$

$S \rightarrow bSY \rightarrow bac \checkmark$

$n_a(w) = 2$

$3n_a(w) = 6$

$w: \boxed{aabb}bb$

$S \rightarrow SS \rightarrow aSX \rightarrow$

$aaSXX \rightarrow aabbbb \checkmark$

5.1

11.

HW

CFG? $L = \{a^n w w^R b^n : w \in \Sigma^*, n \geq 1\}$ $\Sigma = \{a, b\}$

$$\left(\begin{array}{l} ww^R \text{ on } \Sigma = \{a, b\} : n \geq 1 \\ S \rightarrow aSa / bSb / a / b \end{array} \right)$$

$a \checkmark$ $aa \checkmark$
 $b \checkmark$ $aba \checkmark$
 $abba \checkmark$

$$S \rightarrow aSb / W$$

$$W \rightarrow aWa / bWb / a / b$$

Test

aabab : $S \rightarrow aSb \rightarrow aaWab \rightarrow aabab \checkmark$

abab : $S \rightarrow aSb \rightarrow \times$

13.

$$L = \{a^n b^n : n \geq 0\}$$

a) ST L^2 is CFGb) ST L^k is CFG $\forall k \geq 1$ c) ST \mathbb{Z} & L^* are CFG.

$$L^2: a^n b^n a^m b^m$$

$$S \rightarrow AA$$

$$A \rightarrow aAb / \lambda$$

$$L^k: \frac{a^n b^n}{1} \frac{a^m b^m}{2} \dots \frac{a^p b^p}{k}$$

$$S \rightarrow A_1 A_2 \dots A_{k+1}$$

$$A \rightarrow aAb / \lambda$$

$$\mathbb{Z}: \Sigma^* = a^n b^n \rightarrow \text{CFG}$$

$$S \rightarrow SS / aSb / bSa / \lambda$$

$$= \text{CFG}$$

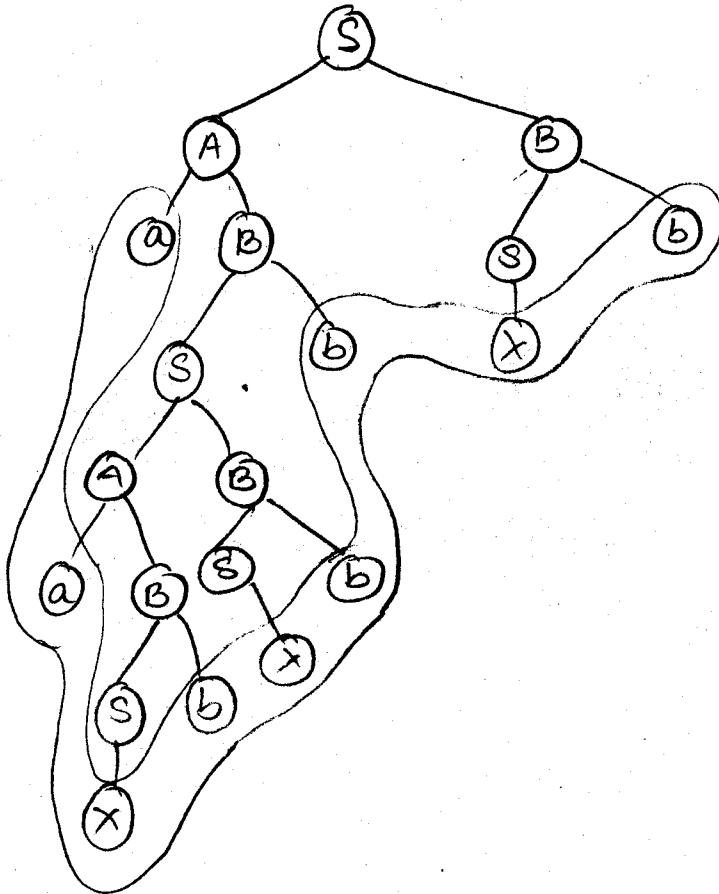
$$= \text{CFG}$$

$$L^*: \lambda \in L, L^0 \text{ CFG}$$

$$L^k \in \text{CFG}$$

$$\therefore L^* \text{ is CFG}$$

19

$$S \rightarrow AB/\lambda$$
$$A \rightarrow aB$$
$$B \rightarrow Sb$$


* **Parsing:** finding a sequence of productions by which $w \in L(G)$ is derived.

Exhaustive search has flaws.

- ① Tedious
- ② it is possible that it never terminates for a $w \notin L(G)$

* **SIMPLE GRAMMAR:**

A context free Grammar $G = (V, T, S, P)$ is said to be a simple Grammar or s-grammar if all productions are of the form

$$\rightarrow A \rightarrow ax.$$

- $A \in V, a \in T, x \in V^*$

\rightarrow Any pair (A, a) occurs at most once in P .

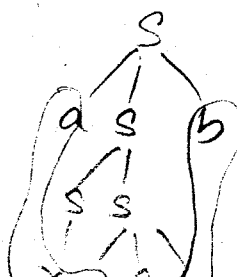
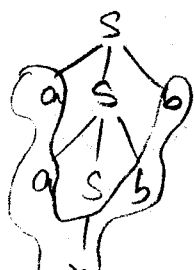
* A CFG is said to be **ambiguous** if there exists some $w \in L(G)$ that has at least two distinct derivation trees

* $S \rightarrow aS / bSS / c$ ✓ S-Grammar
 $\because A \rightarrow ax, (A, a)$ never repeats

$S \rightarrow aS / bSS / aSS / c$ ✗ not S-Grammar
 $\because (A, a)$ repeat though $A \rightarrow ax$

* $S \rightarrow aSb / SS / \lambda$

w: aabb



\therefore ambiguous.

→ One way to resolve ambiguity is

① Associate precedence rules \Rightarrow change semantics

→ Another way is to rewrite the Grammar.

→ If Every Grammar that generates L is ambiguous, then L is called Inherently ambiguous.

→

EXERCISES

①
HW

find an SGrammar for $L(aaa^*b + b)$

aaa^*b .

$S \rightarrow aaAb / b$

$A \rightarrow aA / \lambda$

$S \rightarrow aaAb / b$

$A \rightarrow aA / \lambda$

$A \rightarrow ax$

$(A, a) \times$

$S \rightarrow aA / b$

$A \rightarrow a$

$B \rightarrow aB$

$$(aaa^*b + b) = (aab + b) + aaa^*b$$

\downarrow aa^* \downarrow $\text{min one } a$

aab

$S \rightarrow ax$

$x \rightarrow ay$

$y \rightarrow b$

$aaa^*b + b$

$S \rightarrow ax / b$

$x \rightarrow ay$

$y \rightarrow ay / b$

Test

aab: $S \rightarrow ax \rightarrow aaY \rightarrow aab \checkmark$

aaab: $S \rightarrow ax \rightarrow aaY \rightarrow aaaY \rightarrow aaab \checkmark$

2.
HW

find an sGrammar for $L = \{a^n b^n : n \geq 1\}$

$$\{a^n b^n : n \geq 1\}$$

$$\lambda \notin L(G)$$

$$S \rightarrow aSb / ab$$

$$B \rightarrow b$$

$$S \rightarrow aSB / aB$$

(S,a) X

$$S \rightarrow aB / \lambda$$

$$B \rightarrow S /$$

$$S \rightarrow aA$$

$$A \rightarrow b / aAB$$

$$B \rightarrow b$$

③

find an sGrammar for $L = \{a^n b^{n+1} : n \geq 2\}$

$$a^n b^{n+1} : n \geq 2$$

$$S \rightarrow aSb / aabbb$$

$$a^n b^{n+1} : n \geq 0$$

$$S \rightarrow aSb / b$$

$$n \geq 2$$

Substitute $n=0$.

aabbbb :

$$X \rightarrow aY$$

$$Y \rightarrow aZ$$

$$Z \rightarrow bW$$

$$W \rightarrow bV$$

$$V \rightarrow b$$

$$\underbrace{\quad \quad \quad}_{a * n} \cdot aabbbb \cdot \underbrace{\quad \quad \quad}_{b * n}$$

aabbbb

$$S \rightarrow aA$$

$$A \rightarrow aB$$

$$B \rightarrow bX / aBY$$

$$X \rightarrow bY$$

$$Y \rightarrow b$$

$$\uparrow a^n b^n$$

Test

$$aabbbb: S \rightarrow aA \rightarrow aaB \rightarrow aabX \rightarrow aabbY \rightarrow aabbbb \checkmark$$

$$aaabbbbb: S \rightarrow aA \rightarrow aaB \rightarrow aaaBY \rightarrow aaabXY \rightarrow aaabbbYY \rightarrow aaabbbbb \checkmark$$

6

HW

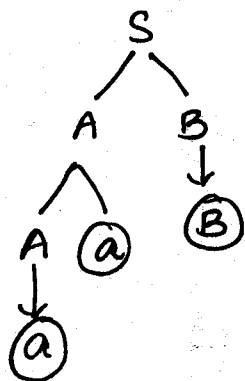
Show that the following Grammar is ambiguous:

$$S \rightarrow AB / aaB$$

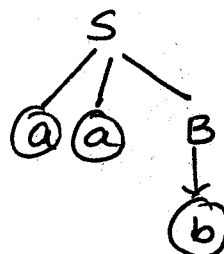
$$A \rightarrow a / Aa$$

$$B \rightarrow b$$

$w = aab$



$w = aab$



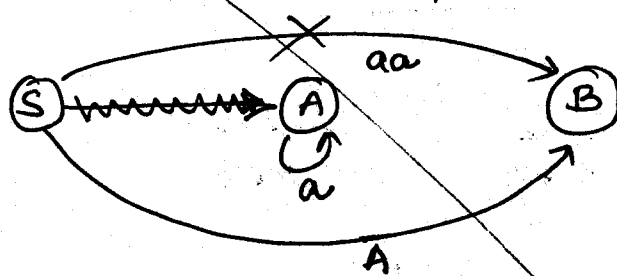
$\exists w = aab$ st \exists two distinct derivation trees as above.
 \therefore The Grammar is AMBIGUOUS.

7

HW

Construct unambiguous grammar for above Grammar.

$S \rightarrow aaB$ is repetitive.



$$\therefore S \rightarrow AB$$

$$A \rightarrow a / Aa$$

$$B \rightarrow b$$

$$a^*b$$

$$S \rightarrow aA$$

$$A \rightarrow b / aA$$

$$X \rightarrow aX / b$$

$$ab$$

$$aab$$

$$aaab$$

$$\Rightarrow a^*b : \{a^n b : n \geq 1\}$$

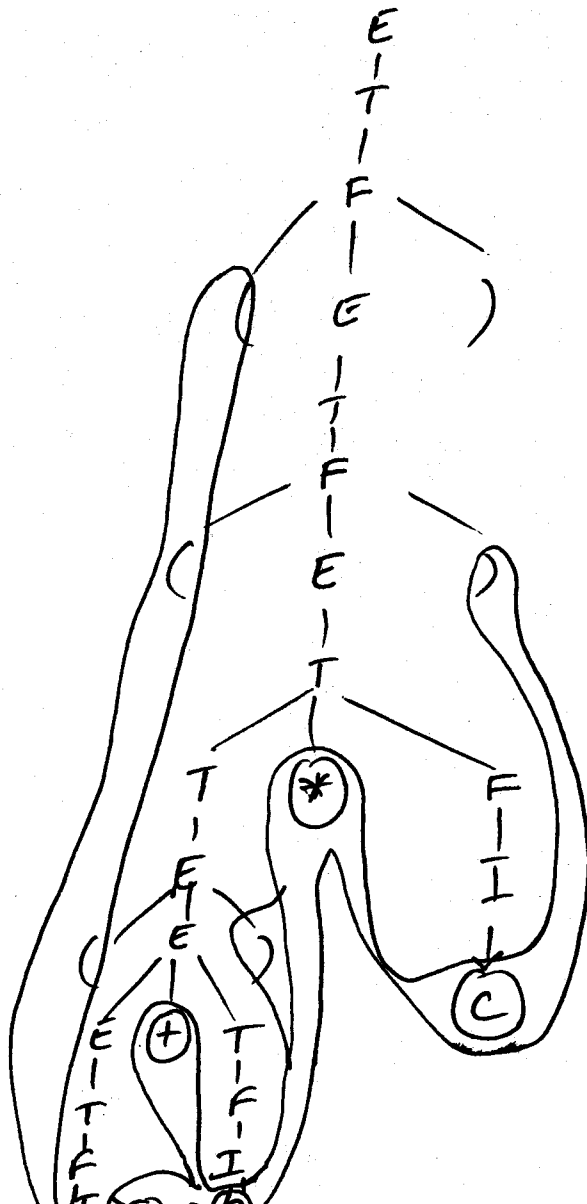
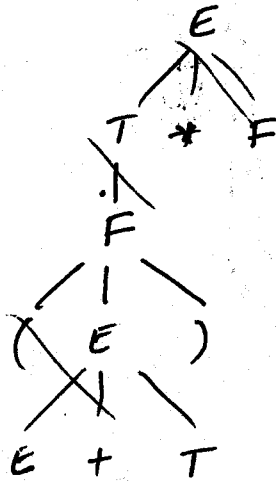
$$S \rightarrow aA / b$$

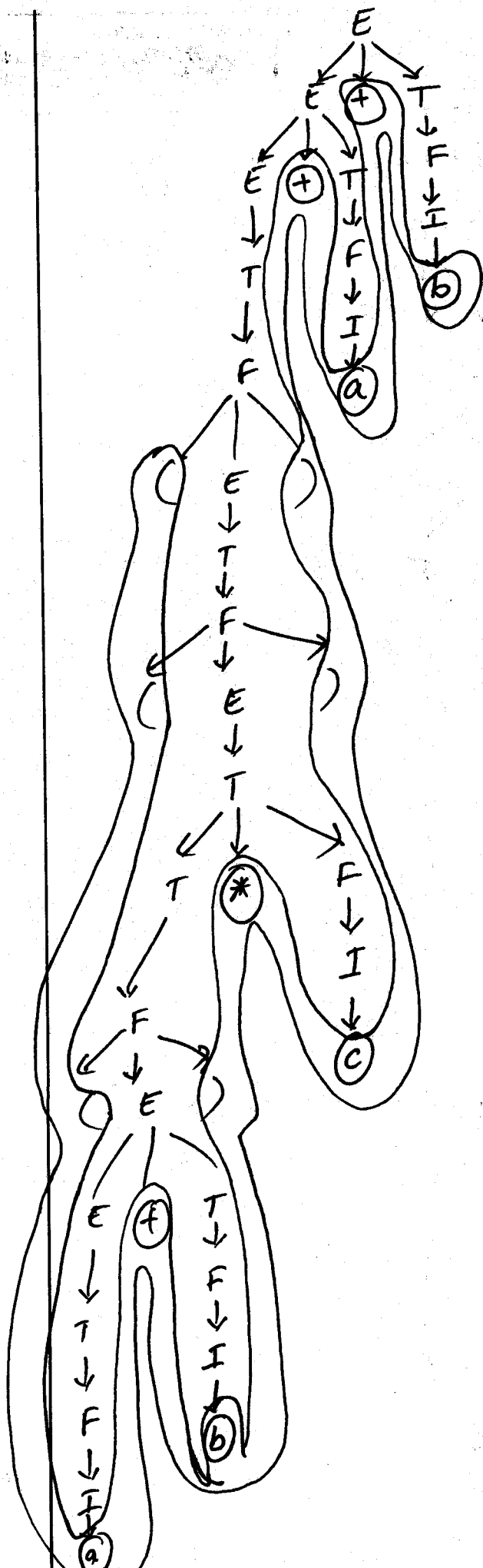
$$S \rightarrow aA$$

$$A \rightarrow aA / b$$

$$S \rightarrow aS / b$$

Give derivation Tree for $((a+b) * c) + a + b$ using

$$I \rightarrow a/b/c$$

$$((a+b)*c)$$



$$((a+b)*c)+a+b$$

⑩ Give unambiguous grammar equivalent to set of all regular expressions on $\Sigma = \{a, b\}$

$\{\lambda, a, b, ab, ba, abb, \dots\}$
 $\downarrow \downarrow \downarrow \downarrow$
 $\cdot SS$

$(S \rightarrow aSb / bSa / SS / a / b / \lambda)$
 ambiguous, strings $\in \{a, b\}^*$
 RE:

$\checkmark S \rightarrow SS / aSb / bSa / a / b$

?

$(a+b)^*$

$S \rightarrow aS / bS / \lambda$

$S \rightarrow aS / bS / a / b$

$S \rightarrow aSX / bSX$

$X \rightarrow a / b$

$ab \checkmark$

$abab \checkmark$

⑪ S.T the language $L = \{ww^R : w \in \{a, b\}^*\}$ is not inherently ambiguous.

ww^R :

$S \rightarrow aSa / bSb / a / b / \lambda$

Test abx $abbaabba \checkmark$ $aa \checkmark$
 $aba \checkmark$ $\lambda \notin L(a)$

all grammars are ambiguous

ww^R

$S \rightarrow aSa / bSb / a / b$

$S \rightarrow aSX / bSY$

$X \rightarrow a$

$Y \rightarrow b$

$S \rightarrow aSa / bSb / a / b / \lambda$

$S \rightarrow aSa / bSb / a / b / aa / bb$

$S \rightarrow aSa / bSb / aa / bb / aaa / bab / aba / bbb$

$S \rightarrow aSa / bSb / aaa / aba / bbb / bab / aa / bb$

① Eliminate λ

② eliminate UNIT-P

Simplification of CFG & Normal forms

Ex: 6.1

$$G = (\{A, B\}, \{a, b, c\}, A, P)$$

$$A \rightarrow a|aaA|abBc$$

$$B \rightarrow abbA|b$$

$$A \rightarrow a|aaA|ababbbAc|abbc$$

Ex: 6.2

$$S \rightarrow A$$

$$A \rightarrow aA|\lambda$$

$$B \rightarrow bA$$

$$S \rightarrow A$$

$$A \rightarrow aA|\lambda$$

$$S \rightarrow A$$

$$A \rightarrow aA|a$$

$$S \rightarrow A$$

$$A \rightarrow aA|a$$

6.3

$$S \rightarrow aS|A|C$$

$$A \rightarrow a$$

$$B \rightarrow aa$$

$$C \rightarrow acb$$

$$S \rightarrow aS|a|C$$

$$C \rightarrow acb$$

$$S \rightarrow aS|a$$

6.4.

$$S \rightarrow aS, b$$

$$S \rightarrow aS, b|\lambda$$

$$S \rightarrow aS, b|ab$$

$$S \rightarrow aS, b|ab$$

6.5

find CFG without λ -productions

$$S \rightarrow ABaC$$

$$A \rightarrow BC$$

$$B \rightarrow b|\lambda$$

$$C \rightarrow D|\lambda$$

$$D \rightarrow d$$

$$① \lambda \notin L(G)$$

$$② V_N : \{A, B, C\}$$

$$S \rightarrow ABaC|BaC|AaC|ABa|aC|Ba|Aa|a$$

$$A \rightarrow BC|B|C$$

$$B \rightarrow b$$

$$C \rightarrow D$$

$$D \rightarrow d$$

Rules to Eliminate λ -Productions

- ① check that $\lambda \notin L(G)$
- ② $V_N = \{ \dots \}$
- ③ Eliminate all λ -productions
- ④ make all combinations of nullable variables.

Rules to eliminate UNIT-Productions

STEP #1: find dependency Graph for unit-Productions.

nodes \rightarrow variable

connections \rightarrow where Unit Production \neq .

STEP #2:

$$S \xRightarrow{*} A$$

$$A \xRightarrow{*} B$$

$$S \xRightarrow{*} B$$

$$B \xRightarrow{*} A$$

STEP #3:

Grammar without
UNIT-Productions

+ make
Extensions

Eg: 6.6

$$S \rightarrow Aa/B$$

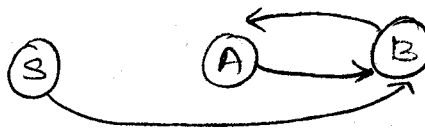
$$B \rightarrow A|bb$$

$$A \rightarrow a|bc/B$$

$$S \rightarrow B$$

$$B \rightarrow A$$

$$A \rightarrow B$$



$$S \xRightarrow{*} A$$

$$A \xRightarrow{*} B$$

$$B \xRightarrow{*} A$$

$$S \xRightarrow{*} B$$

$$S \rightarrow Aa \quad / a/bc/bb$$

$$B \rightarrow bb \quad / a/bc$$

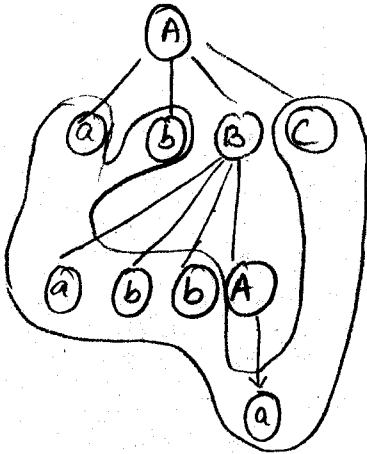
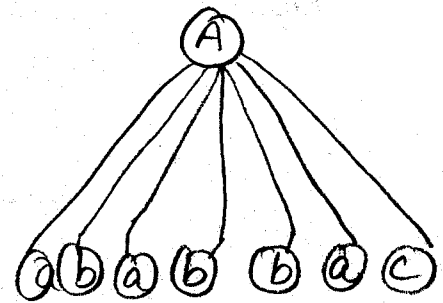
$$A \rightarrow a/bc \quad / bb$$

CH#6.1
(EXERCISES)

$$B \rightarrow abbA / b$$

A → a / aaA / ~~aabbaaa~~
 ababbac / baabbacAc /
~~aabbbb~~ abbc .

Derivation tree for $w = ababbac$?


$$w = ababbac$$
 $w = ababbaac$

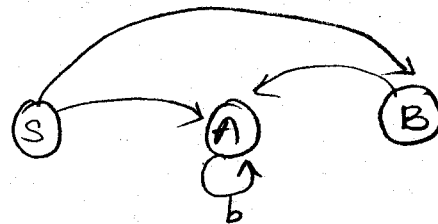
Eliminate all useless Productions for the Grammar.

$$A \rightarrow bA$$
$$B \rightarrow AA$$

Substitution:

$$A \rightarrow bA$$

~~B → AA~~

 ~~$A \rightarrow bA$~~

✓ never ends

↓

never Ends

$$L = \{w = a^\infty b^\infty : w \in \Sigma_{A^*}^*\}$$

HW ⑥

Eliminate Useless Productions from

$$S \rightarrow a/aA/B/C$$

$$A \rightarrow aB/\lambda$$

$$B \rightarrow Aa$$

$$C \rightarrow cCD$$

$$D \rightarrow ddd$$

Substitution:

$$S \rightarrow a/aA/B/cCddd$$

$$A \rightarrow aB/\lambda$$

$$B \rightarrow Aa$$

$$C \rightarrow cCddd$$

$$S \rightarrow a/aA/Aa$$

$$A \rightarrow aAa/\lambda$$

④

Eliminate λ -Productions from

$$S \rightarrow AaB/aaB$$

$$A \rightarrow \lambda$$

$$B \rightarrow bbA/\lambda$$

$$① \lambda \notin L(G)$$

$$② V_N = \{A, B\}$$

$$S \rightarrow AaB/aaB$$

$$B \rightarrow bbA/①$$

$$A \rightarrow \lambda$$

$$S \rightarrow aB/aaB/a/aa$$

$$B \rightarrow bb$$

$$\therefore S \rightarrow aB/aaB/a/aa$$

$$B \rightarrow bb$$

Simplified:-

$$S \rightarrow abb/aabb/a/aa$$

GRIEBACH NORMAL FORM:

- restriction NOT on length of Production
- but on POSITIONS in which terminals & variables can appear

$$A \rightarrow ax$$

$$a \in T \quad x \in V^*$$

- looks similar to S-Grammar
- But no-restriction on (A, a) of Productions.

Eg: 6.9

$$\left. \begin{array}{l} S \rightarrow AB \\ A \rightarrow aA / bB / b \\ B \rightarrow b \end{array} \right\} \begin{array}{l} \text{not GNF} \\ A \rightarrow aX \end{array}$$

$$\begin{array}{l} S \rightarrow aAB / bBB / bB \\ A \rightarrow aA / bB / b \\ B \rightarrow b \end{array} \in \text{GNF.}$$

Eg: 6.10 Convert the Grammar $S \rightarrow abSb / aa$ into GNF.

$$X \rightarrow a$$

$$Y \rightarrow b$$

$$S \rightarrow aYSY / aX$$

$$X \rightarrow a$$

$$Y \rightarrow b$$

for every cfa $G, \lambda \in L(G)$

\exists Equivalent \tilde{G} , in GNF.

EXERCISES -
CH #62

② Convert to CNF

$$S \rightarrow asb/ab$$

$$S \rightarrow XSY/XY$$

$$X \rightarrow a$$

$$Y \rightarrow b$$

CNF

$$A \rightarrow BC$$

$$A \rightarrow a$$

$$S \rightarrow XA/XY$$

$$A \rightarrow SY$$

$$X \rightarrow a$$

$$Y \rightarrow b$$

ECNF

HW ③

Convert to CNF:

$$S \rightarrow aSaA/A$$

$$A \rightarrow abA/b$$

Substitution:

$$S \rightarrow aSaA/abA/b$$

$$A \rightarrow abA/b$$

CNF:

$$A \rightarrow BC$$

$$A \rightarrow a$$

$$S \rightarrow aSXA/aYA/b$$

$$A \rightarrow aYA/b$$

$$X \rightarrow a$$

$$Y \rightarrow b$$

$$S \rightarrow XB/XC/b$$

$$A \rightarrow XC/b$$

$$B \rightarrow SD$$

$$C \rightarrow YA$$

$$D \rightarrow XA$$

$$X \rightarrow a$$

$$Y \rightarrow b$$

HW ④

Convert to CNF

$$S \rightarrow abAB$$

$$A \rightarrow bAB/\lambda$$

$$B \rightarrow BAa/A/\lambda$$

$\lambda \notin L(G)$

λ Elimination

$V_N: \{A, B\}$

$$S \rightarrow abAB/abA/abB$$

$$A \rightarrow bAB/bA/bB$$

$$B \rightarrow BAa/A/Ba/Aa$$

$$S \rightarrow \overline{X}YAB/\overline{X}YA/\overline{X}YB$$

$$A \rightarrow \overline{Y}AB/\overline{Y}A/\overline{Y}B$$

$$B \rightarrow BAX/BX/AX/\overline{Y}AB/\overline{Y}A/\overline{Y}B$$

$$X \rightarrow a$$

$$Y \rightarrow b$$

$$B \rightarrow BAa/Ba/Aa/bAB/bA/bB$$

$$S \rightarrow XCB/XC/XD$$

$$A \rightarrow CB/YA/YB$$

$$B \rightarrow EX/BX/AX/CB/YA/YB$$

$$C \rightarrow YA \quad E \rightarrow BA \quad Y \rightarrow b$$

$$S \rightarrow FB/XC/XD$$

$$A \rightarrow CB/YA/YB$$

$$B \rightarrow EX/BX/AX/CB/YA/YB$$

$$C \rightarrow YA$$

$$D \rightarrow YB$$

$$E \rightarrow BA$$

$$F \rightarrow XC$$

$$X \rightarrow a$$

$$Y \rightarrow b$$

G(CNF)

⑤

Convert to CNF:

$$\lambda \notin (14)$$

$$S \rightarrow AB/aB$$

$$A \rightarrow aab/\lambda$$

$$B \rightarrow bbA$$

 λ -elimination

$$V_N: \{A\}$$

$$S \rightarrow AB/aB/B$$

$$A \rightarrow aab$$

$$B \rightarrow bbA/bb$$

Substitution

$$S \rightarrow AB/aB/bbA/bb$$

$$A \rightarrow aab$$

$$B \rightarrow bbA/bb$$

$$S \rightarrow AB/XB/\underline{XYB}/YY$$

$$A \rightarrow \underline{XXY}$$

$$B \rightarrow \underline{YYA}/YY$$

$$X \rightarrow a \quad Y \rightarrow b$$

$$S \rightarrow AB/XB/CB/YY$$

$$A \rightarrow DY$$

$$B \rightarrow CA/YY$$

$$C \rightarrow YY$$

$$D \rightarrow XX$$

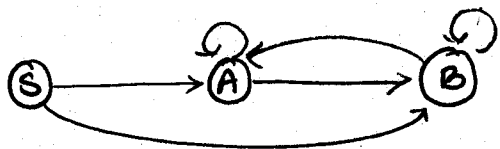
HW 7

Draw dependency Graph for

$$S \rightarrow abAB$$

$$A \rightarrow bAB/\lambda$$

$$B \rightarrow BAa/A/\lambda$$



⑩ Convert to CNF

$$S \rightarrow aSb/bSa/a/b$$

$$\boxed{S \rightarrow aX}$$

CNF

$$S \rightarrow aSY/bSX/a/b$$

$$X \rightarrow a$$

$$Y \rightarrow b$$

⑪ Convert to CNF

$$S \rightarrow aSb/lab$$

$$S \rightarrow aSY/aY$$

$$Y \rightarrow b$$

HW 12

Convert to CNF

$$S \rightarrow ab/aS/aaS$$

$$S \rightarrow aY/aS/aXS$$

$$X \rightarrow a$$

$$Y \rightarrow b$$

⑫ Convert to CNF

Substitution

$$S \rightarrow ABb/a$$

$$S \rightarrow aaABb/BBb/a$$

$$S \rightarrow aaABb/bABb/a$$

$$A \rightarrow aaA/B$$

$$A \rightarrow aaA/bAb$$

$$A \rightarrow aaA/bAb$$

$$B \rightarrow bAb$$

$$B \rightarrow bAb$$

$$B \rightarrow bAb$$

$$S \rightarrow aXABY/bAYBY/a$$

$$A \rightarrow aXA/bAY$$

$$B \rightarrow bAY$$

$$X \rightarrow a$$

*)

Palindrome: CNF = ?

 $\lambda \notin L(G)$ $ww^R \rightarrow \cancel{aba} \cancel{bab} : \text{add } a/b$ $S \rightarrow aSa / bSb / \lambda / a / b$ $S \rightarrow aSa / bSb / aa / bb$ $S \rightarrow XSX / YSY / XX / YY / a / b$ $X \rightarrow a$ $Y \rightarrow b$ $S \rightarrow XA / YB / XX / YY / a / b$ $A \rightarrow SX$ $X \rightarrow a$ $Y \rightarrow b$ $B \rightarrow SY$

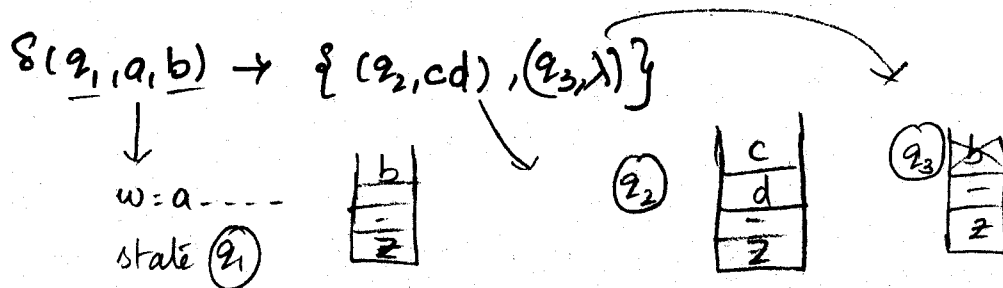
*

 $L = \{a^n : n \geq 1\}$ CNF = ? $S \rightarrow aaaaS / aaaa$ $S \rightarrow AAAAS / AAAA$ $A \rightarrow a$ $S \rightarrow XXS / XX$ $X \rightarrow AA$ $A \rightarrow a$ $S \rightarrow YS / XX$ $X \rightarrow AA$ $Y \rightarrow XX$ $A \rightarrow a$

Npda: Nondeterministic Pushdown Automata:

$$M = (Q, \Sigma, \Gamma, \delta, q_0, z, F)$$

Ex: 7.1



Ex: 7.2

$$Q = \{q_0, q_1, q_2, q_3\}$$

$$\Sigma = \{a, b\}$$

$$\Gamma = \{0, 1\}$$

$$z = 0$$

$$F = \{q_3\}$$

$$\delta(q_0, a, 0) = \{(q_1, 10), (q_3, \lambda)\}$$

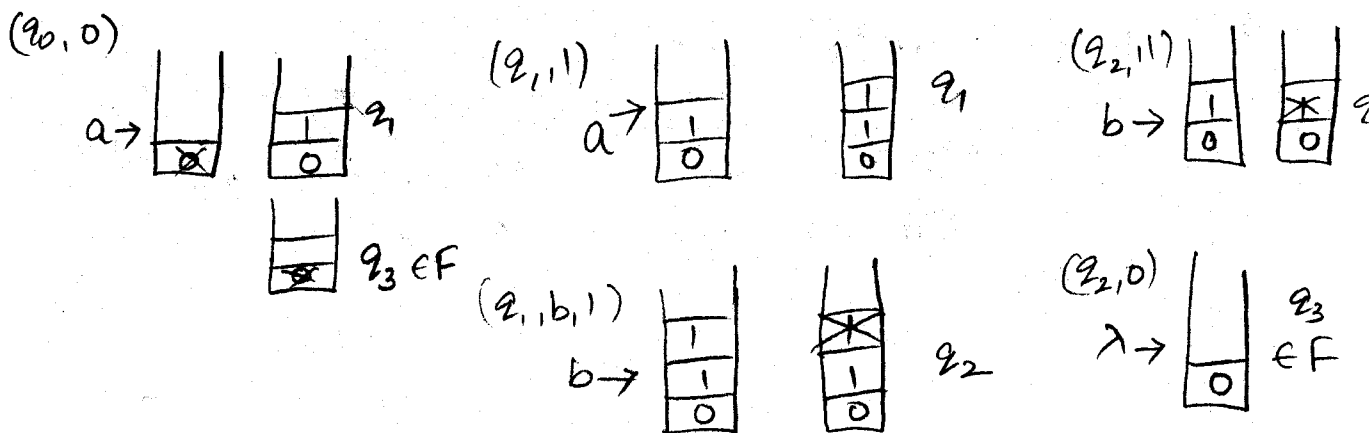
$$\delta(q_0, \lambda, 0) = \{(q_3, \lambda)\}$$

$$\delta(q_1, a, 1) = \{(q_1, 11)\}$$

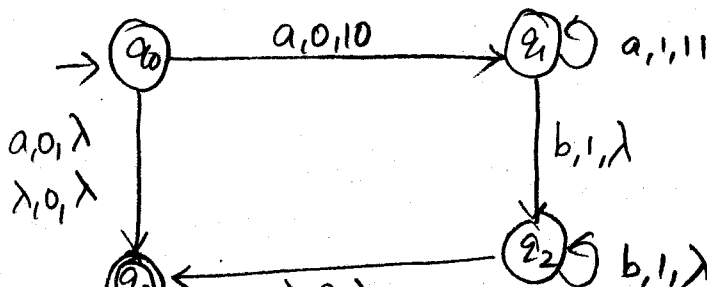
$$\delta(q_1, b, 1) = \{(q_2, \lambda)\}$$

$$\delta(q_2, b, 1) = \{(q_2, \lambda)\}$$

$$\delta(q_2, \lambda, 0) = \{(q_3, \lambda)\}$$



$$a \cup \{a^n b^n : n \geq 0\}$$



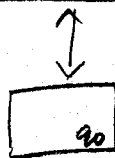
class

Prathima Bhima
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DATE:

$$L = \{a^n b^n a^n : n \geq 0\}$$

{ | □ | a | a | a | b | b | b | a | a | a | □ }



$$\begin{aligned} \delta(q_0, a) &= q_1 \\ \delta(q_0, b) &= q_2 \\ \delta(q_0, \square) &= q_0 \end{aligned}$$

~~aa bb a~~

~~aa bba~~

~~abb~~

$$\delta(q_0, a) = (q_1, \square, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, b) = (q_2, b, R)$$

$$\delta(q_2, b) = (q_2, b, R)$$

$$\delta(q_2, a) = (q_3, a, L)$$

$$\delta(q_3, b) = (q_3, a, R)$$

$$\delta(q_3, a) = (q_3, a, R)$$

$$\delta(q_3, \square) = (q_4, \square, L)$$

$$\delta(q_4, a) = (q_5, \square, L)$$

$$\delta(q_5, a) = (q_6, \square, L)$$

$$\delta(q_6, a) = (q_7, a, L)$$

$$\delta(q_7, a) = (q_7, a, L)$$

$$\delta(q_7, b) = (q_7, b, L)$$

$$\delta(q_7, \square) = (q_0, \square, R)$$

$$\delta(q_6, \square) = (q_6, \square, R)$$

$$\delta(q_0, \square) = (q_6, \square, R)$$

not a

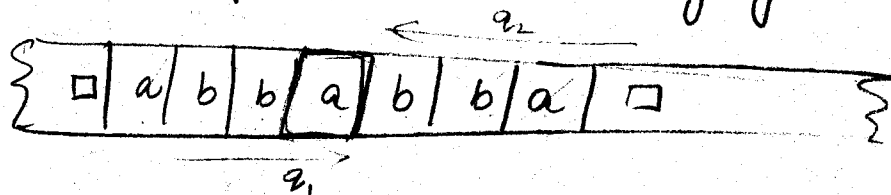
$M = ($

Test

$w = aba : \in L$

$q_0 aba \vdash \square q_1 ba \vdash b q_2 a \vdash q_3 ba \vdash a q_3 a \vdash$
 $aa q_3 \square \vdash a q_4 a \vdash q_5 a \vdash q_6 \square \vdash q_f \square$
 \downarrow
accepted

* Design TM that accepts PALINDROME language.



L/R/R

$$\delta(q_0, a) = (q_1, \square, R)$$

$$\delta(q_0, b) = (q_2, \square, R)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_2, b) = (q_2, b, R)$$

$$\delta(q_1, b) = (q_1, b, R)$$

$$\delta(q_2, a) = (q_2, a, R)$$

$$\delta(q_1, \square) = (q_3, \square, L)$$

$$\delta(q_2, \square) = (q_5, \square, L)$$

$$\delta(q_3, a) = (q_4, \square, L)$$

$$\delta(q_5, b) = (q_4, \square, L)$$

$$\delta(q_4, a) = (q_4, a, L)$$

$$\delta(q_4, b) = (q_4, b, L)$$

$$\delta(q_4, \square) = (q_0, \square, R)$$

$$\delta(q_0, \square) = (q_6, \square, R)$$

even string

$$\delta(q_5, \square) = (q_6, \square, R)$$

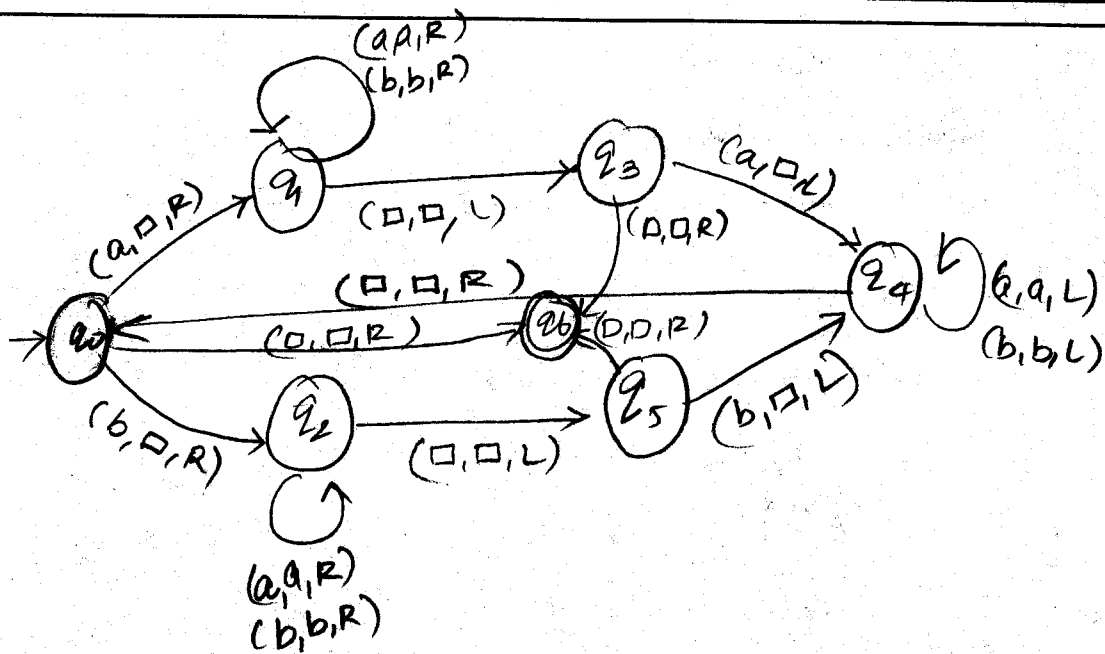
$$\delta(q_3, \square) = (q_6, \square, R)$$

} odd string
middle = a/b

M: (-)

Test: ababa

instead of expecting another
a/b to delete, if no symbol/
 \Rightarrow accepted.



Using Machine as Transducer:

rejected strings of acceptor = \bar{L}

$$\hat{w} = f(w)$$

$$\boxed{q_0 w \xrightarrow{*}_{TM} q_f \hat{w}} \quad (q_f \in F)$$

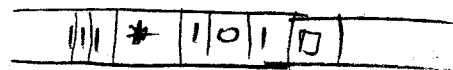
→ Computable function: \leftrightarrow has TM.

→ q_i ends @ finite no. of steps

→ whatever the complexity.

\approx Algorithm, whatever the complexity.

* Addition with TM



11
10

use unary NS:

Addition

{ 1 1 1 0 1 1 1 1 }

- skip till spl. char
- replace it with 1 & move to L
- till \square move to L & del last 1.

$$\delta(q_0, 1) = (q_0, 1, R)$$

$$\delta(q_0, 0) = (q_1, 1, R)$$

$$\delta(q_1, 1) = (q_1, 1, R)$$

$$\delta(q_1, \square) = (q_2, \square, L)$$

$$\delta(q_2, 1) = (q_3, 0, L)$$

$$\delta(q_3, 1) = (q_3, 1, L)$$

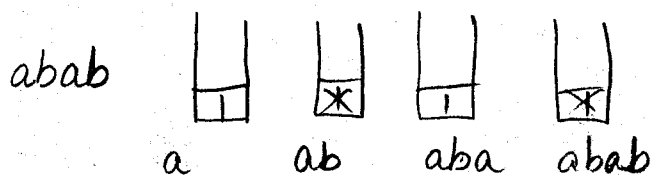
$$\delta(q_3, \square) = (q_4, \square, R)$$

Every TM has to have R/W @ beginning.

Ex 7.4

Construct an npda for the language

$$L = \{w \in \{a,b\}^* : n_a(w) = n_b(w)\}$$



$$\delta(q_0, \lambda, z) = \{(q_f, z)\}$$

$$\delta(q_0, a, z) = \{(q_0, 1z)\}$$

$$\delta(q_0, b, z) = \{(q_0, 0z)\}$$

$$\delta(q_0, a, 1) = \{(q_0, 11)\}$$

$$\delta(q_0, b, 0) = \{(q_0, 00)\}$$

$$\delta(q_0, a, 0) = \{(q_0, \lambda)\}$$

$$\delta(q_0, b, 1) = \{(q_0, \lambda)\}$$

$$M = (\dots)$$

Test: $w = abab$

$$(q_0, abab, z) \vdash (q_0, bab, 1z) \vdash (q_0, ab, 1z) \vdash (q_0, b, 11z) \vdash$$

$$(q_0, \lambda, z) \vdash (q_f, \lambda, z)$$

$\downarrow \in F$ accepted

$w = bbaa$

$$(q_0, bbaa, z) \vdash (q_0, baa, 0z) \vdash (q_0, aa, 00z) \vdash (q_0, a, 01z) \vdash$$

$$(q_0, \lambda, z) \vdash (q_f, \lambda, z)$$

\downarrow
 $\in F$

accepted.

Ex. 7.5 Construct an npda for $L = \{ww^R : w \in \{a,b\}^+\}$

$$\delta(q_0, a, z) = \{(q_0, az)\}$$

$$\delta(q_0, b, z) = \{(q_0, bz)\}$$

$$\delta(q_0, a, a) = \{(q_0, aa)\}$$

$$\delta(q_0, b, b) = \{(q_0, bb)\}$$

$$\delta(q_0, a, b) = \{(q_0, ab)\}$$

$$\delta(q_0, b, a) = \{(q_0, ba)\}$$

$$\delta(q_1, a, a) = \{(q_1, \lambda)\}$$

$$\delta(q_1, b, b) = \{(q_1, \lambda)\}$$

$$\delta(q_1, \lambda, z) = \{(q_f, z)\}$$

$$\delta(q_0, a, a) = \{(q_1, a)\}$$

$$\delta(q_0, b, b) = \{(q_1, b)\}$$

Test: $w = abba$

$$(q_0, abba, z) \vdash (q_0, bba, az) \vdash (q_0, ba, bazz) \vdash$$

$$(q_1, ba, bazz) \vdash (q_1, a, azz) \vdash (q_1, \lambda, z) \vdash (q_f, \lambda, z)$$

accepted

$w = abab$

$$(q_0, abab, z) \vdash (q_0, bab, az) \vdash (q_0, ab, bazz) \vdash (q_0, b, abazz) \vdash (q_0, \lambda, babazz)$$

rejected

$$M = (\{q_0, q_1, q_f\}, \{a, b\}, \{a, b, z\}, \delta, z, q_f)$$

② Construct npda's that accept the following Regular languages

(a) $L = L(aaa^*b)$

$$\delta(q_0, a, z) = \{(q_1, az)\}$$

$$\delta(q_1, a, a) = \{(q_2, aa)\}$$

$$\delta(q_2, a, a) = \{(q_2, aa)\}$$

$$\delta(q_2, b, a) = \{(q_f, ba)\}$$

$$w = aab$$

$$(q_0, \underline{aab}, \underline{z}) \vdash (q_1, \underline{ab}, \underline{az}) \vdash (q_2, \underline{b}, \underline{aaaz})$$

$$\vdash (q_f, \underline{baaaz})$$

↓

ϵF

accepted

(b) $L_2 = L(aab^*aba^*)$

$$\delta(q_0, a, z) = \{(q_1, az)\}$$

$$\delta(q_1, a, a) = \{(q_2, aa)\}$$

$$\delta(q_2, b, a) = \{(q_2, ba)\}$$

$$\delta(q_2, b, b) = \{(q_2, bb)\}$$

$$\delta(q_2, a, a) = \{(q_3, aa)\}$$

$$\delta(q_2, a, b) = \{(q_3, ab)\}$$

$$\delta(q_3, b, a) = \{(q_f, ba)\}$$

$$\delta(q_f, a, b) = \{(q_f, ab)\}$$

$$\delta(q_f, a, a) = \{(q_f, aa)\}$$

$$\delta(q_f, \lambda, b) = \{(q_f, b)\}$$

$$\delta(q_f, \lambda, a) = \{(q_f, a)\}$$

$$w = aab$$

$$(q_0, \underline{aab}, \underline{z}) \vdash (q_1, \underline{ab}, \underline{az}) \vdash (q_2, \underline{ab}, \underline{aa})$$

$$\vdash (q_3, \underline{b}, \underline{aaa}) \vdash (q_f, \underline{\lambda}, \underline{baaa}) \vdash$$

$$(q_f, \underline{\lambda}, \underline{baaa})$$

↓

ϵF

accepted

M(---)

(c) $L_{UL_2} : (aaa^*b) \cup (aab^*aba^*)$

$$\delta(q_0, a, z) = \{q_1, az\}$$

$$\delta(q_1, a, a) = \{q_2, aa\}$$

$$\delta(q_2, a, a) = \{q_3, aa\} \quad a^+ \quad aaa^*$$

$$\delta(q_2, b, a) = \{q_4, ba\}, \{q_6, ba\} \quad \boxed{aab \text{ final}}, \boxed{aaa^*b \text{ final}}$$

$$\delta(q_3, b, a) = \{q_4, ba\}$$

$$\delta(q_3, b, b) = \{q_3, bb\}$$

$$\delta(q_3, a, b) = \{q_4, ab\}$$

$$\delta(q_4, b, a) = \{q_5, ba\} \quad \boxed{aab^*ab \text{ final}}$$

$$\delta(q_5, a, a) = \{q_6, aa\} \quad \boxed{aab^*aba^+ \text{ final}}$$

$$\delta(q_6, \lambda, z) = \{q_6, \lambda\}$$

Test:

aabab:

$$\delta(q_0, aabab, z) \vdash \delta(q_1, abab, az) \vdash \delta(q_2, bab, aaz) \vdash \boxed{\delta(q_6, \lambda, baaz)}$$

$$\vdash \delta(q_4, ab, baaz) \vdash$$

store

$npda \Leftrightarrow CFG$

$CFG \rightarrow npda$

$CFG \rightarrow GNF \rightarrow npda$



$A \rightarrow aX$

$$\delta(q_0, \lambda, z) = (q, sz)$$

$$\delta(q, a, A) = (q, x)$$

$$\delta(q, \lambda, z) = (q_f, \lambda)$$

76.

Ex:

$S \rightarrow asbb/a$

$\left. \begin{array}{l} S \rightarrow asYY/a \\ Y \rightarrow b \end{array} \right\} GNF$

$$\delta(q_0, \lambda, z) = \{(q_1, sz)\}$$

$$\delta(q_1, a, s) = \{(q_1, sYY), (q_1, \lambda)\}$$

$$\delta(q_1, \lambda, z) = \{(q_2, \lambda)\}$$

$$\delta(q_1, b, Y) = \{(q_1, \lambda)\}$$

$M = (\dots)$

Test

Ex: 7.7.

$$S \rightarrow aA$$

$$A \rightarrow aABC / bB / a$$

$$B \rightarrow b$$

$$C \rightarrow c$$

npda = ?

$$\rightarrow \delta(q_0, \lambda, z) = \{(q_1, sz)\}$$

$$\delta(q_1, a, S) = \{(q_1, A)\}$$

$$\delta(q_1, a, A) = \{(q_1, ABC), (q_1, \lambda)\}$$

$$\delta(q_1, b, A) = \{(q_1, B)\}$$

$$\delta(q_1, b, B) = \{(q_1, \lambda)\}$$

$$\delta(q_1, c, C) = \{(q_1, \lambda)\}$$

$$\rightarrow \delta(q_1, \lambda, z) = \{(q_f, \lambda)\}$$

EXERCISES

①
②
③

$$S \rightarrow aABB / aAA$$

$$A \rightarrow aBB / a$$

$$B \rightarrow bBB / A$$

npda = ?

$$\delta(q_0, \lambda, z) = \{(q_1, sz)\}$$

$$\delta(q_1, a, S) = \{(q_1, ABB), (q_1, AA)\}$$

$$\delta(q_1, a, A) = \{(q_1, BB), (q_1, \lambda)\}$$

$$\delta(q_1, b, B) = \{(q_1, BB), (q_1, A)\}$$

$$\delta(q_1, \lambda, z) = \{(q_f, \lambda)\}$$

(a)

find npda with 2 states for $L = \{a^n b^{n+1} : n \geq 0\}$

$$S \rightarrow aSb / b$$

$$\text{GNF: } S \rightarrow aSY / b$$

$$Y \rightarrow b$$

$$\delta(q_0, \lambda, z) = \{(q_1, sz)\}$$

$$\delta(q_1, a, S) = \{(q_1, SY)\}$$

$$\delta(q_1, b, Y) = \{(q_1, \lambda)\}$$

$$\delta(q_1, \lambda, z) = \{(q_b, \lambda)\}$$

$$\delta(q_1, b, S) = \{(q_1, \lambda)\}$$

$$\delta(q_0, \lambda, z) = \{(q_0, sz_1)\}$$

$$\delta(q_0, a, S) = \{(q_0, SY)\}$$

$$\delta(q_0, b, S) = \{(q_0, \lambda)\}$$

$$\delta(q_0, b, Y) = \{(q_0, \lambda)\}$$

$$\delta(q_0, \lambda, z_1) = \{(q_b, \lambda)\}$$

(b)

find npda with 2 states that accepts $L = \{a^n b^{2n} : n \geq 1\}$

$$S \rightarrow aSbb / \lambda$$

GNF:

$$S \rightarrow aSbb / aBB$$

$$S \rightarrow aSBB / aBB$$

$$B \rightarrow b$$

$$\delta(q_0, \lambda, z) = \{(q_0, sz_1)\}$$

$$\delta(q_0, a, S) = \{(q_0, SBB), (q_0, BB)\}$$

$$\delta(q_0, b, B) = \{(q_0, \lambda)\}$$

$$\delta(q_0, \lambda, z_1) = \{(q_b, \lambda)\}$$

Apda : DCFL

Apda① $\delta(q, a, b)$ contains at most one element② if $\delta(q, \lambda, b) \neq \emptyset$ then $\delta(q, c, b) = \emptyset \quad \forall c \in \Sigma$

Eg: 7.10

 $L = \{a^n b^n : n \geq 0\}$ is DCFL.

dpda = ?

$$\delta(q_0, a, 0) = \{(q_1, 10)\}$$

$$\delta(q_1, a, 1) = \{(q_1, 11)\}$$

$$\delta(q_1, b, 1) = \{(q_2, \lambda)\}$$

$$\delta(q_2, b, 1) = \{(q_2, \lambda)\}$$

$$\delta(q_2, \lambda, 0) = \{(q_0, \lambda)\}$$

 $q_0 \in F$

*)

$$L = \{a^n b^n : n \geq 0\} \cup \{a\}$$

dpda = ?

$$\delta(q_0, a, z) = (q_1, az)$$

$$\{q_0, q_1, q_2\} \in F$$

$$\delta(q_1, a, a) = (q_1, aa)$$

$$\delta(q_1, b, a) = (q_1, \lambda)$$

$$\delta(q_1, \lambda, z) = (q_1, \lambda)$$

$$\delta(q_1, a, a) = (q_1, aa)$$

$$\delta(q_1, b, a) = (q_2, \lambda)$$

$$\delta(q_2, b, a) = (q_2, \lambda)$$

①

ST. $L = \{a^n b^n : n \geq 0\}$ is a DCFL.

abb

$$\delta(q_0, \lambda, z) = (q_0, \lambda)$$

$$\delta(q_0, a, z) = (q_1, 1z)$$

$$\delta(q_1, a, 1) = (q_1, 11)$$

$$\delta(q_1, b, 1) = (q_1, \lambda)$$

$$\delta(q_1, \lambda, z) = (q_0, \lambda)$$

dpda

 \therefore DCFL

Test abb: accepted

$$\delta(q_0, \underline{a} \underline{b} \underline{b}, z) \vdash \delta(q_1, \underline{b} \underline{b}, 1z) \vdash \delta(q_1, b, 1z) \vdash \delta(q_1, \lambda, z) \vdash (q_0, \lambda)$$

aabbbb: accepted

$$\delta(q_0, \underline{a} \underline{a} \underline{b} \underline{b} \underline{b} \underline{b}, z) \vdash \delta(q_1, \underline{a} \underline{b} \underline{b}, 1z) \vdash \delta(q_1, \underline{b} \underline{b}, 111z) \vdash \delta(q_1, b \underline{b} \underline{b}, - - -)$$

③

$L = \{a^n b^n : n \geq 1\} \cup \{b\}$ DCFL?

$$\delta(q_0, a, z) = (q_1, 1z)$$

$$\delta(q_0, b, z) = (q_0, \lambda)$$

$$\delta(q_1, a, 1) = (q_1, 11)$$

$$\delta(q_1, b, 1) = (q_1, \lambda)$$

$$\delta(q_1, \lambda, z) = (q_0, \lambda)$$

= dpda

 \therefore DCFL ✓

5
8

$L = \{a^n b^m : n=m \text{ or } n=m+2\}$ is DCFL?

$$\{a^n b^n\} \cup \{a^{n+2} b^n\}$$

9

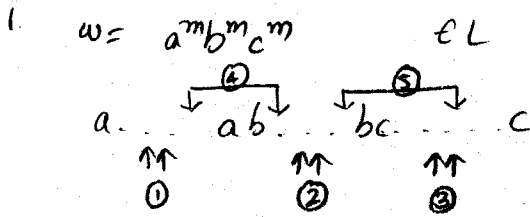
$w c w^R$

↓

start matching.

Properties of CFL

Ex: 1.1 $L = \{a^n b^n c^n : n \geq 0\}$ is not context free.



2.1 $v = a$

$y = a$

$w_i = a^{m+2i-2} b^m c^m$

$i > 1 \Rightarrow m+2i-2 > m$

$w_i \notin L$

$$\therefore n_a(w_i) \neq n_b(w_i) \\ \neq n_c(w_i)$$

$w_i \notin L$

2.2 $v = b$

$y = b$

2.3 $v = c$

$y = c$

2.4 $v = a$

$y = b$

2.5 $v = b$

$y = c$

$w_i = a^{m+i-1} b^{m+i-1} c^m$

$i > 1 \Rightarrow m+i-1 > m$

$\Rightarrow n_a(w_i) \neq n_b(w_i)$

$n_b(w_i) \neq n_c(w_i)$

$w_i \notin L$

Similar case

similar cases

\therefore as Pumping lemma fails, L is not a CFL.

Ex: 1.2

$$L = \{ww : w \in \{a,b\}^*\} \text{ CFL?}$$

1. $w = a^n b^n a^m b^m$

2.1 $w_i = a^{n+2i-2} b^n a^m b^m$

$i > 1 \Rightarrow n+2i-2 > n$

$w_i \notin L$

$$v = a \quad \{ \text{first } w \}$$

$$\begin{cases} 2.2. v = b, y = b \\ 2.3. v = a, y = a \\ 2.4. v = b, y = b \end{cases}$$

similar cases

2.5 $v = a, y = b$

$w_i = a^{n+i-1} b^{n+i-1} a^m b^m$

$$i > 1 \Rightarrow n_a(\text{first } w) > \\ n_a(\text{second } w)$$

$\therefore w_i \notin L$

2.6 $v = b, y = c$

2.7 $v = a, y = b$

similar case

L is not CFL as PL fails.

Ex 8.3 ST. $L = \{a^n! : n \geq 0\}$ is not context free.

1. $w = a^{m!} \in L$

2. $v = a^k$

$y = a^l$

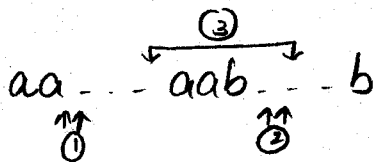
$w_p = a^{(m-(k+l)+2)!}$

$k+l < m : m-(k+l) > 0$

$\therefore m-(k+l) > m!$

\therefore not CFL.

Ex 8.4 ST $L = \{a^n b^j : n = j^2\}$ is not CFL.



1. $w = a^{m^2} b^m \in L$

2.1 $u = a$
 $y = a$

$w_p = a^{m^2+2i-1} b^m$

$i > 1 \Rightarrow m^2+2i-1 > m^2$

$\therefore w_p \notin L$

(\approx 2.2 $u=b, y=b$)

2.3 $u=a$

$y=b$

$w_p = a^{m^2+i-1} b^{m+i-1}$

$i=0 : m^2-1 \neq (m-1)^2$

$w_p \notin L$

$\therefore L$ is not CFL

4) $L = \{a^m b^j a^j b^n : m, j > 0\}$ is CFG or not?

$\delta(q_0, a, z) = \{(q_0, az)\}$

$\delta(q_0, a, a) = \{(q_0, aa)\}$

$\delta(q_0, b, a) = \{(q_1, ba)\}$

$\delta(q_1, b, b) = \{(q_1, bbb)\}$

$\delta(q_1, a, b) = \{(q_2, \lambda)\}$

$\delta(q_2, a, b) = \{(q_2, \lambda)\}$

$\delta(q_2, b, a) = \{(q_3, \lambda)\}$

$\delta(q_3, b, a) = \{(q_3, \lambda)\}$

$\delta(q_3, \lambda, z) = \{(q_4, \lambda)\}$

$$L = \{a^n b^m c^j : n \leq j\}$$

$$1. w = a^m b^m c^{m+1} \in L$$

2.1

$$u = a$$

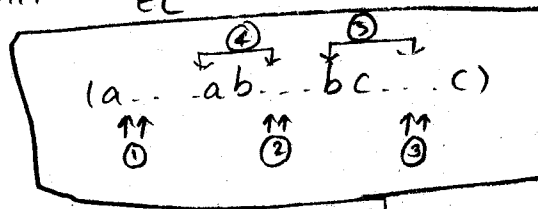
$$y = a$$

$$w_i = a^{m+2i-2} b^m c^{m+1}$$

$$i > 1 \quad m+2i-2 > m$$

$$\Rightarrow n_a(w) \neq n_b(w)$$

$$w_i \notin L$$



$$2.4 \quad u = a$$

$$y = b$$

$$w_i = a^{m+i-1} b^{m+i-1} c^{m+1}$$

$$i > 2 \Rightarrow m+i-1 > m+1$$

$$\therefore w_i \notin L$$

$$n_c(w) \neq n_a(w)$$

$$> n_b(w)$$

2.5

2.2

similar cases

$$2.3 \quad u = c$$

$$y = c$$

$$w_i = a^m b^m c^{m+2i-2}$$

$$i = 0 \quad m-1 \leq m$$

$$w_i \notin L$$

$$\therefore L \notin CFL$$

EXERCISES

②

ST $L = \{a^n : n \text{ is a prime no.}\}$ is not CFL.

$$1. w = a^m$$

$$m \text{ is prime} : \in L$$

2.

$$a \dots a$$

$$\uparrow \uparrow$$

$$u = a$$

$$y = a$$

$$w_i = a^{m+2i-2}$$

$$i = 0$$

$$m+2i-2 = m-2$$

$m-2$ is not prime

$$i > 0$$

$$m+2i-2$$

$$i < 0$$

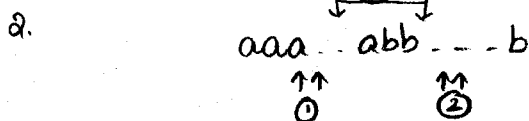
$$m+2i-2$$

} not necessarily prime

$$\therefore PL \text{ fails} \Rightarrow \text{NOT CFL}$$

⑤ Is $L = \{a^n b^m : n = 2^m\}$ CFL?

1. $w = a^{2^m} b^m \in L$



2.1 $\begin{aligned} &v = a \\ &y = a \\ &w_i = a^{2^m + 2^i - 2} b^m \end{aligned}$

2.2 similar for $v = b, y = b$

2.3 $\begin{aligned} &v = a \\ &y = b \\ &w_i = a^{2^m + i - 1} b^{m + i - 1} \end{aligned}$

$\boxed{2^m + 2^i - 2}$

$i = 0 : 2^m - 2 \neq 2^m$

ie $\text{pow}(a) \neq 2^m$

$\therefore w_i \notin L$

$i = 0 : a^{2^m - 1} b^{m - 1}$

$2^{m-1} = 2^m - 2 = 2^{m-1} - 1$

$\boxed{2^{m-1} > 2^{m-1}}$

ie $\text{pow}(a) \neq 2^m$

$\therefore w \notin L$

is not CFL:

⑦ a) $L = \{a^n b^j : n \leq j^2\}$

$w = a^{j^2} b^j$

not CFL

b) $L = \{a^n b^j : n \geq (j-1)^3\}$

$w = a^{(j-1)^3} b^j$

not CFG

c) $n_a(w) \geq n_b(w) \leq n_c(w)$

$a^n b^{n+1} c^{n+2}$

⑧ CFL or not?

(a) $L = \{a^n w w^R a^n : n \geq 0, w \in \{ab\}^*\}$

$\begin{aligned} &w w^R : \text{CFL} \\ &a^n : \text{CFL} \end{aligned} \left. \begin{array}{l} \text{CFL closed under} \\ \text{concatenation} \end{array} \right\} = \boxed{\text{CFL}}$

$\delta(q_2, a, b) = \{(q_3, \lambda)\}$

$\delta(q_3, a, a) = \{(q_3, \lambda)\}$

$\delta(q_2, \lambda, a) = \{(q_1, \lambda)\}$

$\delta(q_0, a, z) = \{(q_0, az), (q_1, az)\}$

$\delta(q_0, a, a) = \{(q_0, aa), (q_1, aa)\}$

$\delta(q_1, a, a) = \{(q_1, aa)\}$

$\delta(q_1, b, a) = \{(q_1, ba)\}$

$\delta(q_1, a, b) = \{(q_1, ab), (q_2, \lambda)\}$

$\delta(q_1, b, b) = \{(q_1, bb), (q_2, \lambda)\}$

$\delta(q_2, a, a) = \{(q_2, \lambda)\}$

$\delta(q_2, b, b) = \{(q_2, \lambda)\}$

CH# 8.1

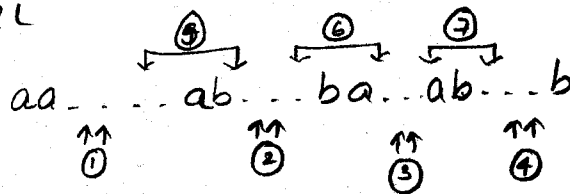
⑧

⑥.

$$L = \{a^n b^j a^n b^j : n \geq 0, j \geq 0\}$$

not CFG

$$1. w = a^m b^k a^m b^k \in L$$



$$\begin{array}{|c|} \hline 2.1 \\ \hline 2.2 \\ \hline 2.3 \\ \hline 2.4 \\ \hline \end{array} \quad \begin{array}{l} v=a \\ y=a \\ w_i = a^{m+2i-2} b^k a^m b^k \end{array}$$

$$i > 0 : m+2i-2 > m$$

$$w_i \notin L$$

$$\begin{array}{|c|} \hline 2.5 \\ \hline 2.6 \\ \hline 2.7 \\ \hline \end{array} \quad \begin{array}{l} v=a \\ y=b \\ w_i = a^{m+i-1} b^{m+i-1} a^m b^m \end{array}$$

$$i=0 : m-1 \neq m$$

$$w_i \notin L$$

PL fails \Rightarrow NOT CFL

⑨

$$L = \{a^n b^j a^j b^n : n \geq 0, j \geq 0\}$$

$$\delta \left(\begin{array}{|c|} \hline a^n b^j \\ \hline \end{array}, \begin{array}{l} \delta(q_0, a, z) = \{ (q_0, az) \} \\ \delta(q_0, a, a) = \{ (q_0, aa) \} \\ \delta(q_0, b, a) = \{ (q_1, ba) \} \end{array} \right.$$

$$\delta(q_0, b, z) = \delta(q_1, bz)$$

$$\begin{array}{|c|} \hline a^0 b^j \\ \hline \end{array}$$

$$\delta(q_1, b, b) = \{ (q_2, bb) (q_3, \lambda) \}$$

$$\delta(q_2, a, b) = \{ (q_1, \lambda) \}$$

$$\delta(q_2, b, b) = \{ (q_3, \lambda) \}$$

$$\delta(q_3, b, b) = \{ (q_3, \lambda) \}$$

$$\delta(q_3, \lambda, z) = \{ (q_1, \lambda) \}$$

$$\delta(q_0, \lambda, z) = \{ (q_0, \lambda) \}$$

$$\longrightarrow \lambda \in L(q)$$

$$\begin{array}{|c|} \hline b^j a^j \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline b^0 a^0 \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline a^n \times b^n \\ \hline \end{array}$$

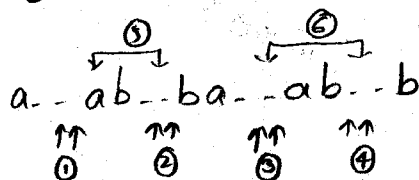
 \therefore CFL

①

$$L = \{a^n b^j a^k b^l : n+j < k+l\}$$

1. $w = a^n b^n a^m b^m$

$n < m$



2.1

$U = a$

$y = a$

$w_i = a^{n+2i-2} b^n a^m b^m$

$i > 0 \quad n+2i > m$

but $n < m$

$\therefore w_i \notin L$

2.5

$U = a$

$y = b$

$w_i = a^{n+i-1} b^{n+i-1} a^m b^m$

pow(LHS)

pow(RHS)

$2n+2i-2 : 2m$

$n+i-1 : m$

$n < m$

$i > 0 \quad n+i > m$

$w_i \notin L$

is PL false, NOT CFL

②

$$L = \{a^n b^j a^k b^l : n \leq k, j \leq l\}$$

NOT CFL

③

$$L = \{a^n b^n c^j : n \leq j\}$$

NOT CFL

④

$$L = \{w \in \{a,b,c\}^* : n_a(w) = n_b(w) = 2n_c(w)\}$$

NOT CFL.

CFL closed under \cup union
 \cdot concatenation
 \rightarrow $*$ Star Closure.

$$RL \cap CFL = CFL$$

NOT closed under \rightarrow Intersection \cap
 \rightarrow Complement \bar{A} :

Ex: 8.7 ST $L = \{a^n b^n : n \geq 0, n \neq 100\}$ is CFL.

$$L_2 = \{a^n b^n : n \geq 0\}$$

$$L_1 = \{a^n b^n : n = 100\}$$

$$L_1 = \{a^{100} b^{100}\} \rightarrow \text{Regular}$$

Regular languages are closed under complement
 $\therefore L_1$ is also Regular.

$$L = L_2 \cap \bar{L}_1 = \{a^n b^n : n \neq 100, n \geq 0\}$$

$$\downarrow \quad \downarrow$$

$$CFL \quad RL$$

$\therefore L$ is a CFL.

Ex: 8.8 ST $L = \{a^n b^n c^n : n \geq 0\}$ is not CFL.

PL fails: also: $L_1 = (a^* b^* c^*) \rightarrow \text{Regular}$

we know $L_2 = \{a^n b^n c^n\}$ is NOT CFL.

$$L \cap L_1 = L_2$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$\text{Regular} \quad \text{NOT CFL}$$

Case (i) L is CFL $\Rightarrow L_2$ should be CFL, but is not $\Rightarrow L$ is NOT CFL
 Case (ii) L is not CFL $\Rightarrow L$ is not CFL. True.

Is Empty / Is not Empty

$N \rightarrow \epsilon$
 $N \rightarrow X$

Q:

$S \rightarrow XY$
 $X \rightarrow AX$
 $X \rightarrow AA$
 $A \rightarrow a$
 $Y \rightarrow BY$
 $Y \rightarrow BB$
 $B \rightarrow b$

$S \rightarrow XY$
 ~~$X \rightarrow aX$~~
 $X \rightarrow aa$
 ~~$Y \rightarrow bY$~~
 $Y \rightarrow bb$

$S \rightarrow aabb$

$L(a)$ NOT empty

Q:

$S \rightarrow XY$
 $X \rightarrow AX$
 $A \rightarrow a$
 $Y \rightarrow BY$
 $Y \rightarrow BB$
 $B \rightarrow b$

$S \rightarrow XY$
 $X \rightarrow aX$
 ~~$Y \rightarrow bY$~~
 $Y \rightarrow bb$

$S \rightarrow Xbb$
 $X \rightarrow aX$

$L(a)$
Is Empty.

Q*)

$S \rightarrow XS / YZ$

$X \rightarrow YX$
 $Y \rightarrow YY$
 $Y \rightarrow XX$
 $X \rightarrow A$
 $Z \rightarrow SX$

Is Empty

Q*)

$S \rightarrow AB$
 ~~$A \rightarrow BS$~~
 ~~$B \rightarrow AS$~~
 ~~$A \rightarrow CC$~~
 ~~$B \rightarrow CC$~~
 ~~$C \rightarrow SS$~~
 ~~$A \rightarrow a/b$~~

$S \rightarrow aB / bB$
 ~~$B \rightarrow aaS / bbS / abS / baS$~~
 $B \rightarrow bb / bbb / bbbb$

$S \rightarrow abb / abbb / abbbb / bbb / bbbb / bbbbb$

$L(a)$ Not Empty

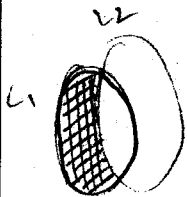
CFL * CFL₂ not closed
RL - RL is closed

$$\text{ST } L_1 - L_2 = \text{CFL}$$

$$\text{if } L_1 : \text{CFL} \neq \\ L_2 : \text{RL}.$$

$L_1 - L_2$: assume closed under difference. - ①

$$L_1 - L_2 = L_1 \cap \bar{L}_2$$



① \Rightarrow LHS is CFL

RHS is not CFL as ^{CFL} languages are not closed under concatenation

\therefore assumption of ① is wrong

\therefore CFL not closed under '-'

$$L_1 = \text{CFL}$$

$$L_2 = \text{RL}$$

$$L_1 - L_2 = L_1 \cap \bar{L}_2$$

$L_1 \cap \bar{L}_2$: context free language.
 $\downarrow \quad \downarrow$
 CFL RL

\Rightarrow If $L_1 : \text{CFL}$, $L_2 : \text{RL}$ then
 closed under difference.

② $\text{ST } \text{CFL}$ not closed under \cup & \cap

DCFL \Rightarrow DPDA

$$L_1, L_2 \in \text{DCFL}$$

$L = L_1 \cup L_2 \Rightarrow \boxed{S \rightarrow S_1 / S_2} \rightarrow \text{non-deterministic}$
 $S \rightarrow \epsilon$ not DCFL

$$L_1 \cap L_2 \Rightarrow \overline{\overline{L_1} \cup \overline{L_2}} = \overline{\bar{L}} \text{ not DCFL}$$

(18)

SI $L = \{w \in \{a,b\}^* : n_a(w) = n_b(w) : w \text{ don't contain substring } aab\}$

$$L_2 = (a+b)^* aab (a+b)^*$$

↓
Regular language $\Rightarrow \bar{L}_2$ also RL

$$L_1 = \{ \{a,b\}^* : n_a(w) = n_b(w) \}$$

we know NOT CFL
(PL fails)

$a^m b^k a^l b^j$
$(m+l = k+j)$

$$L \cap \bar{L}_2 = L$$

↓
RL

Case (i) L is CFL $\Rightarrow L_1$ is CFL / not true

Case (ii) L is NOT CFL $\Rightarrow L_1$ is NOT CFL / true

$\therefore L$ is not CFL